This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

An Efficient and Incentive-Compatible Mechanism for Energy Storage Markets

Bharadwaj Satchidanandan and Munther A. Dahleh

***Abstract*—A key obstacle to increasing renewable energy penetration in the power grid is the lack of utility-scale storage capacity. Transportation electrification has the potential to overcome this obstacle since Electric Vehicles (EVs) that are not in transit can provide battery storage as a service to the grid. This is referred to as EV-Power grid integration, and could potentially be a key milestone in the pathway to decarbonize the electricity and the transportation sectors. We first show that if EV-Power grid integration is not done carefully, then contrary to improving the cost efficiency of operating the grid, it could in fact be counterproductive to it. This occurs due to two phenomena operating in tandem — the randomness of EV usage patterns and the possibility of strategic behavior by EV users. We present a market-based solution to address this issue. Specifically, we develop a mechanism for energy storage markets using which the system operator can efficiently integrate a fleet of strategic EVs with random usage patterns into the grid, utilize them for storage, and satisfy the demand at the minimum possible cost.**

***Index Terms*—Energy storage markets, Electric Vehicles, Stochastic valuations, Incentive compatible mechanism.**

I. Introduction

A major impediment to high renewable energy penetration in the power grid is the scarcity of energy storage capacity in the grid. Utility-scale battery storage is expensive at current technology, and so any energy that is generated must be consumed immediately. This paradigm could change substantially with increased Electric Vehicle (EV) penetration since EVs that are not in transit can provide battery storage as a service to the grid. Prior studies estimate that on an average, a car is parked for more than 95% of the time [1], indicating the huge potential for EVs to double as energy storage resources in the grid. To illustrate the potential of EV energy storage, take the example of the state of Massachusetts. It consumes an average of 146GWh of electric energy per day [2]. On the other hand, the battery capacity of a Tesla Model S EV is about 100kWh. This implies that about 1.4 million EVs possess enough battery capacity to power Massachusetts for an entire day. This amounts to less than 64% of the vehicles registered in Massachusetts today [3]. The situation is similar in most other parts of the US and the world, indicating that even moderate levels of EV penetration could provide significant storage capacity.

The time periods during which an EV can lease its battery to the grid are private knowledge of the EV user and is unknown to the Independent System Operator (ISO). However, the ISO requires this information to optimally operate the grid, or more precisely, to determine the optimal power dispatch of the generators and the optimal storage schedule of the EVs. Consequently, the ISO requests the EV users to report in the day-ahead market the time periods during which they can lease their battery the following day. However, this brings forth two challenges that need to be addressed.

The first challenge is that the travel times of people are in general random, and so the EV users may not precisely know in the day-ahead market the time periods during which they can lease their EV batteries the following day. Rather, they may know these time periods only with some uncertainty. To account for this, we model the time periods during which an EV can lease its battery as a random quantity, and require that the EV users only report the probability distribution of this quantity in the day-ahead market.

The second challenge is that the EV users could be strategic, and so they may not report the aforementioned probability distribution truthfully. As we elaborate in Section III, each EV user has associated with it a utility function, and the EV users may bid strategically to maximize their own respective utilities. Moreover, having bid some probability distribution in the day-ahead market, an EV may misreport its usage pattern in real time if there is possibility for it to obtain a higher utility by doing so than by reporting it truthfully. Such behavior could potentially be counterproductive to the cost- and energy-efficient operation of the grid as illustrated by the following example.

Example 1. *Suppose that a day consists of two time periods, and suppose that the demand sequence* $\mathbf{d}$ *of the load in these time periods is* $\mathbf{d} = \{0, 1\}$*. Let the production function* c_g *of the generator be such that* $c_g(\{1, 0\}) = 0$ *and* $c_g(\{0, 1\}) = 2$*. That is, it costs the generator* 0 *to produce* 1J *of energy at time period* 1 *and* 0J *of energy at time period* 2*, and so on. The cost* c_g *for all other* 2*-tuples is infinite. We suppose that this generator has a low ramping rate – a characteristic that is typical of high-efficiency generators – and so its power dispatch must be scheduled well in advance of the time of power delivery. Specifically, its power dispatch must be scheduled in the day-ahead market.*

The system also consists of a reserve generator which has a high ramping rate which can produce energy in the spot market to balance real-time demand-supply mismatches. Let the production function c_s *of the reserves be such that* $c_s(\{0, 0\}) = 0$ *and* $c_s(\{0, 1\}) = 11$*. The cost* c_s *for all other* 2*-tuples is infinite.*

Suppose that there is only one EV in the system with a battery capacity of 1J*. As we elaborate in Section III, the*

The authors are with the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139, USA (Email: bsatchid@mit.edu, dahleh@mit.edu)

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

usage pattern of an EV on any given day is specified by a quantity known as its "deadline" on that day. An EV's deadline on a given day is defined as the time period until which the EV can lease its battery to the grid on that day. Then, the usage pattern of an EV being random is equivalent to its deadline being random. Suppose that the EV's deadline takes the value 1 *with probability* p *and the value* 2 *with probability* $1-p$.

As elaborated in Section III, each EV has a cost function associated with it. Suppose that the cost incurred by an EV is equal to the negative of the net energy injected into it during the time that it is connected to the grid.

Now, the ISO is confronted with two options to meet the demand. The first option is for it to schedule the generator to produce the energy sequence $\mathbf{g} = \{0, 1\}$ *and use it to serve the load. This results in a total cost of meeting the demand — defined as the sum of the costs incurred by the generator, the EV, and the reserves — to be equal to* 2.

The second option is to schedule the generator to produce the energy sequence $\mathbf{g} = \{1, 0\}$ *and store the energy generated in the first time step in the EV. If the EV remains connected to the grid in the second time step, then the ISO discharges it to satisfy the demand, resulting in a total cost of* 0. *On the other hand, if the EV disconnects at time step* 1, *then the ISO purchases* 1J *in the spot market at time step* 2 *at cost* $c_s(\{0, 1\}) = 11$ *to satisfy the demand, thereby resulting in a total cost of* 10. *Hence, if the EV disconnects at its true deadline, then the total cost of meeting the demand equals* 0 *with probability* $1 - p$ *and equals* 10 *with probability* p. *Hence, the total expected cost of meeting the demand if the ISO decides on the second option is equal to* $10p$.

Now, the goal of the ISO, as we elaborate in Section III, is to minimize the total expected cost of meeting the demand, and so it must choose the first option if $2 \leq 10p$ *and the second option if* $2 > 10p$. *However, the difficulty is that the ISO does not know the value of* p, *and must rely only on the value* $\widehat{p}$ *reported by the EV in the day-ahead market in order to make the decision. The EV bidding* $\widehat{p} = p$ *is not a dominant strategy. To see this, suppose that* $p = 0.21$. *If the EV bids* $\widehat{p} = 0.21$, *then the ISO would decide on the first option, and so the cost incurred by the EV would be equal to* 0. *On the other hand, if the EV bids* $\widehat{p} = 0.19$, *then that causes the ISO to decide on the second option, thereby resulting in the EV being charged with* 1J *in the first time step. If the EV then disconnects at its deadline, then it would exit the system with a charge of* 1J *with probability* 0.21 *and a charge of* 0J *with probability* 0.79. *This would result in it incurring an average cost of* -0.21, *which is lesser than the average cost of* 0 *that it would incur if it bids* p *truthfully. However, the total expected cost of meeting the demand as a result of the EV's false bid is equal to* 2.1, *which is greater than the total cost of* 2 *that would result if the ISO simply decides to never utilize the EV for storage.*

The above example illustrates a scenario wherein the EV can lower its average cost by misreporting only its deadline distribution. However, the EV could also misreport its deadline realization to lower its cost. To see this, consider the case when $p = 0.19$ *and suppose that the EV reports* $\widehat{p} = p$ *truthfully in the day-ahead market. It follows from the above discussion that the ISO would decide on the second option. Now, if the EV reports its deadline truthfully in real time, then the expected cost that it would incur is equal to* -0.19. *On the other hand, by misreporting its deadline realization to be equal to time step* 1, *the EV can exit the system with* 1J *of charge, thereby resulting in it incurring a lower cost of* -1. *Since the ISO is not privy to the EV's deadline realization, it cannot ascertain if the EV disconnects at the first time step due to its deadline arriving at that time or due to strategic behavior. The total expected cost of meeting the demand as a result of the EV's false report would equal* 10, *which is greater than both the total expected cost of* 1.9 *that would result if the EV reports its deadline realization truthfully, and also the total cost of* 2 *that would result if the ISO simply decides to never utilize the EV for storage.*

The above example illustrates how strategic EV behavior not only defeats the purpose of utilizing EVs as energy storage units, but could also be counterproductive to the cost- and energy-efficient operation of the grid since it could potentially result in real-time supply shortages which in turn increases the ISO's dependence on the expensive and energy-inefficient reserves. Therefore, if EVs are to be efficiently integrated for storage, it is imperative to devise incentive structures that drive EV users towards truthful behaviors.

At first look, it may appear that simply compensating EVs in the day-ahead market based on the expected duration that they would remain connected to the grid the following day and penalizing them for real-time shortfalls might suffice. However, the problem with such an approach is that an EV may commit some amount in the day-ahead market in good faith but due to the inherent uncertainties in its usage pattern, it may genuinely be unable to meet its commitment in real-time, and thereby get penalized. If the uncertainties are not properly accounted for by the mechanism and the penalties are not carefully designed, then even honest EV users may incur large penalties, thereby discouraging them from voluntarily participating in the market. Hence, it is necessary to devise mechanisms that (i) incentivize the EVs report the probability distribution of their usage patterns truthfully in the day-ahead market, so that the ISO can optimally plan the power dispatch and storage schedules, (ii) incentivize the EVs to remain plugged into the grid until their actual deadlines, so that there are no untoward supply shortages in real time, and (iii) incentivize honest EVs to voluntarily participate in the market. In this paper, we develop a mechanism that achieves all of these objectives.

At a high level, the mechanism consists of a decision rule that specifies an optimal power dispatch sequence of the generator and an optimal energy storage policy for each EV as a function of the deadline distributions that the EVs report in the day-ahead market, and a payment rule that incentivizes EVs to report their deadline distributions *and* their deadline realizations truthfully. The payment rule consists of two components — (i) a "day-ahead payment" that reflects the expected cost savings in operating the grid due to the storage opportunity that the EVs are expected to provide as per their

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

reported deadline distributions, and (ii) a carefully designed "end-of-the-day settlement" that adjusts the transfers meted out to each EV based on a fairly detailed comparison of what it had reported in the day-ahead market and the actual departure profiles of the EVs in real time. One of the functionalities of the end-of-the-day settlement is to penalize EVs for deviations of their empirically observed behavioral patterns from what is expected as per the probability distributions that they report in the day-ahead market. This is made precise in Section IV. We show how the composite payment rule renders truthful bidding in both the day-ahead market and in real time a dominant strategy for every EV, thereby enabling the ISO to satisfy the demand at minimum possible cost. To the best of our knowledge, we are unaware of any other work that addresses the problem of integrating a fleet of *strategic* EVs with *random usage patterns* into the grid and utilizing them optimally for ancillary services.

The rest of the paper is organized as follows. Section II presents an overview of related work and summarizes the novel aspects of the paper. Section III formulates the mechanism design problem. Section IV describes the proposed mechanism for energy storage markets and establishes the incentive and the efficiency properties guaranteed by it. Section V presents some numerical results that illustrate the cost benefits that EV battery storage service – unreliable though it may be – offers the system. Section VI outlines some potential extensions and concludes the paper.

Notation: Given a vector $\mathbf{x}$, we denote by $x(i)$ the i^{th} component of $\mathbf{x}$. Given a vector $\mathbf{x}(\boldsymbol{\theta})$ which is a function of the variable $\boldsymbol{\theta}$, we denote its i^{th} component by $x(i;\boldsymbol{\theta})$. Given a vector $\mathbf{x}$, we denote by $\mathbf{x}_{-i}$ the vector $\mathbf{x}$ with its i^{th} component removed, and by $[y,\mathbf{x}_{-i}]$ the vector whose i^{th} component is y and the other components are $\mathbf{x}_{-i}$. Given a sequence $\{\mathbf{x}(1),\mathbf{x}(2),\ldots\}$, we use $\mathbf{x}^l$ to denote the $l-$length sequence $\{\mathbf{x}(1),\ldots,\mathbf{x}(l)\}$ and $\mathbf{x}^\infty$ to denote the entire sequence.

II. Related Work

Technologies that utilize EVs as energy storage resources are broadly referred to as vehicle-to-grid technologies, and a large body of literature exists on this topic. A feasibility study of vehicle-to-grid systems is presented in [4], a framework for vehicle-to-grid implementation is described in [5]. Among other aspects, [5] recognizes the need for incentive mechanisms to ensure adequate participation of EV users.

Reference [6] considers a setting where different EVs have to be charged within a known deadline and they have different private valuations per unit energy. A mechanism to elicit the valuations truthfully is presented. Reference [7] addresses a problem where a charging network operator owns a network of charging stations and EV users have different travel routes and charging rate preferences that are private to them. The network operator assigns EVs to charging stations that are on their travel routes based on their reported preferences. A pricing-cum-routing policy to incentivize EVs to reveal their true preferences is presented. Reference [8] views the problem of charging strategic EVs with privately known deadlines and energy valuations as one of designing incentive-compatible mechanisms for multi-unit demand in an online fashion, and develops a mechanism that incentivizes the EVs to report their private parameters truthfully. Reference [9] considers the problem of scheduling the charging of a fleet of EVs which possess certain privately-known valuation functions, identifies certain drawbacks of the VCG mechanism, and proposes two extensions thereof to promote truth-telling. Reference [10] models the interaction between an energy provider and EVs as a Stackelberg game and presents a distributed algorithm for the players to reach a Nash equilibrium.

All of the aforementioned papers view an EV only as a *deferrable load* whose energy demand can be satisfied at any time within a stipulated duration. On the other hand, the viewpoint that we adopt is to view EVs *not only* as deferrable loads, but *also as energy storage resources* that can help reduce the operational cost of the grid. Consequently, a mechanism that is developed within this paradigm rewards EVs not only for the flexibility that they provide in satisfying their energy demand, but also the cost reductions that they afford the ISO by providing energy storage service. Our prior works [11], [12] also adopt this paradigm but [11] restricts attention to a setting where there is no stochasticity in EV deadlines and [12] restricts attention to a scenario where there is no strategic behavior by EV users.

More importantly, most of the existing papers in the literature, including our own prior works [11], [12], fail to model at least one of the following aspects of EVs: (i) the stochasticity of EV deadlines, (ii) the storage opportunity provided by EVs, and (iii) the possibility of strategic behavior by EV users. As mentioned in Section I, the combination of these aspects provides EVs significant leeway for strategic behavior — so much so that the problem of designing an incentive compatible mechanism for this setting appears to be outside the purview of existing results in mechanism design theory. In this paper, we model all of these aspects and address them in a holistic manner.

To the best of our knowledge, the works that are the closest to the problem that we address are [13]–[15] which develop two-stage mechanisms for selling wind power. Although the context of this paper is different from that of [13], [14], the mathematical abstractions of the problems appear to have certain similarities. Specifically, our setting as well as the setting considered in [13], [14] have the property that the valuation of the market participants is a random variable. Both mechanisms consist of two stages over which the final decision is made, and both mechanisms rely on having two settlements in their payment rules. Where our formulation differs from [13], [14] is in a key assumption regarding the realization of the players' valuations. Specifically, while [13], [14] assume the realizations of the players' valuations to be public knowledge, *we assume that the valuation realizations are also private to the players* and that they could bid them strategically in the second stage. The second important departure of our formulation from that in [13], [14] is that the latter consider a "one-shot game" and develop a mechanism that is Incentive-Compatible (IC) and Individually Rational (IR) in expectation whereas we consider a repeated game and seek a mechanism that is IC and IR in a time-averaged sense. It is not clear whether a

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

"one-shot mechanism" that is IC and IR in expectation can be directly extended to repeated games to obtain IC and IR in a time-averaged sense. Simply instantiating the one-shot mechanism in every repetition of the game for example may not guarantee these properties since the players could adapt their bids at any given instance to their observations from all past instances, thereby introducing dependencies between the bids, outcomes, and payments across instances. The aspect of repeated play is a key feature of real world electricity markets, and to the best of our knowledge, we are unaware of any other work in the literature that addresses such a setting.

III. Problem Formulation

The core aspects that impart flavor to the problem that we address are the *stochasticity of EV usage patterns* and the *possibility of strategic behavior* by EV users. It is precisely the former aspect that precludes the use of standard results from mechanism design theory to design energy storage markets for EVs. Therefore, in order to examine in isolation the complexities that arise from these two phenomena, we simplify the problem in several other respects. Specifically, we develop a stylized model which retains only the minimal complexity necessary to exhibit the two aforementioned aspects. Examples of such simplifications include the restriction to a single-bus system, the assumption that the usage pattern of an EV is described by a single parameter known as its "deadline," the assumption that the deadline of each EV is independent and identically distributed across days, the assumption that all EVs are plugged into the grid at the beginning of a day, etc. As it will be apparent shortly, even for such a simplified model, designing an efficient and incentive-compatible energy storage market is nontrivial in that it appears to lie outside the purview of existing results in mechanism design theory. Moreover, the mechanism developed in the context of this simple model contains elements that readily generalize to more complex scenarios, and could therefore potentially guide the design of practical energy storage markets.

Consider a single-bus power system with a single generator, a single load, and n_s storage units or EVs. In addition to EVs, the storage units could also include devices such as Powerwalls [16] that individual households and firms could have installed. We divide time into days, and divide a day into T time intervals. Denote by $d_l(t)$ the energy demand of the load on the l^{th} day at time t, $l \in \mathbb{Z}_+$ and $t \in \{1, \ldots, T\}$. The demand sequence $\mathbf{d}_l := \left[d_l(1), \ldots, d_l(T)\right]$ is a random variable and can typically be forecast in the day-ahead market to an accuracy of within 5% [17]. However, in order to minimize clutter and expose the main ideas clearly, we assume that it is known exactly in the day-ahead market, and furthermore, that it remains the same on all days. Consequently, we drop the subscript l and denote the demand sequence simply as $\mathbf{d} = [d(1), \ldots, d(T)]$.

A. EV Energy Demand

In the day-ahead market corresponding to any day, every EV has an energy demand for its transportation needs on the following day. We model an EV i's energy demand on day l using an "energy valuation function" $v_{i,l} : \mathbb{R} \to \mathbb{R}$ which specifies the EV's valuation for being charged with various energy levels by the time it disconnects from the grid. An EV user can express preferences such as the minimum amount of charge required at its deadline using the energy valuation function. Specifically, by setting the energy valuation function to take "small" values for energy levels lower than what is required for the EV's upcoming trip, and "large" values for those higher than that required for its upcoming trip, the EV user can inform the ISO of its minimum desired charge.

Since the energy requirement of an EV could be different on different days, its energy valuation function could vary across days. However, for ease of exposition, we assume that the energy valuation function of an EV remains the same on all days. Consequently, we drop the subscript "l" and denote simply by v_i the energy valuation function of EV i on any day l.

Every EV is required to report its energy valuation function to the ISO in the day-ahead market, and we assume that they do so truthfully. A subsequent paper addresses the scenario wherein the EVs could misreport their energy valuations. We wish to emphasize at this juncture that it is only the energy valuation function that we assume the EVs to report truthfully; *not their valuations* (or more precisely, costs). The latter, as mentioned before and as we will see shortly, is a random variable whose distribution as well as realizations are private knowledge of the EVs, and they could misreport either or both of these quantities.

B. EV Deadlines

On any day l, $l \in \mathbb{Z}_+$, the time intervals in which an EV i can lease its battery to the grid are characterized by a parameter $\delta_i(l) \in \{1, \ldots, T\}$ called EV i's "deadline" on day l. An EV i on day l is said to have deadline $\delta_i(l)$ if it can lease its battery to the grid at all times lesser than or equal to $\delta_i(l)$ and incurs a large cost J_m for remaining connected beyond time $\delta_i(l)$, thereby missing its deadline. During the time that it is connected to the grid, the ISO can utilize the EV as an energy storage device.

1) Deadline distributions: Owing to the stochasticity of EV usage patterns, we model the deadline sequence $\{\delta_i(1), \delta_i(2), \ldots, \}$ as Independent and Identically Distributed (IID) random variables taking values from $\{1, \ldots, T\}$. Hence, the set of distributions that $\delta_i(1)$ can assume is parameterized by the $\mathbb{R}^T$- dimensional probability simplex $\overline{\Theta}$. We denote by $\theta_i \in \overline{\Theta}$ the parameter vector corresponding to EV i's deadline distribution, and by $\mathbb{P}_{\theta_i}$ the probability distribution of the deadline. That is, for any $t \in \{1, \ldots, T\}$, the quantity $\mathbb{P}_{\theta_i}(t)$ denotes the probability that $\delta_i(1)$ equals t.

While in general, θ_i could take any value in $\overline{\Theta}$, we assume for certain technical reasons that will become clear later that there exists $\epsilon_\theta > 0$ such that for all $i \in \{1, \ldots, n_s\}$ and all $t \in \{1, \ldots, T\}$, $\mathbb{P}_{\theta_i}(t) \geq \epsilon_\theta$. Consequently, given ϵ_θ, we define the set

$$\Theta := \{\theta \in \overline{\Theta} : \mathbb{P}_\theta(t) \geq \epsilon_\theta \ \text{ for all } \ t \in \{1, \ldots, T\}\}$$

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

so that for all $i \in \{1, \ldots, n_s\}$,

$$\theta_i \in \Theta. \tag{1}$$

The deadlines of different EVs are assumed to be independent so that the joint distribution of the EVs' deadlines on any given day is $\mathbb{P}_{\theta_1} \times \ldots \times \mathbb{P}_{\theta_{n_s}}$. We define $\boldsymbol{\theta} := [\theta_1, \ldots, \theta_{n_s}]$.

2) Deadline realizations: We suppose that for every $i \in \{1, \ldots, n_s\}$ and every $l \in \mathbb{Z}_+$, the realization of $\delta_i(l)$ is drawn "by nature" at the beginning of day l according to $\mathbb{P}_{\theta_i}$ and is revealed to EV i at the beginning of day l. In particular, the realization of $\delta_i(l)$ is unknown in the day-ahead market corresponding to day l.

C. Cost functions

The generator, the reserves, and the EVs have associated with them certain cost functions. We describe these one by one.

1) Cost function of the Generator: We denote by $c_g : \mathbb{R}^T_{\geq 0} \to \mathbb{R}$ the production function of the generator so that $c_g(\mathbf{g})$ is the cost incurred by the generator for producing the energy sequence $\mathbf{g} = \big[g(1), \ldots, g(T)\big]$ on any given day, where $g(t)$ denotes the amount of energy produced at time t.

2) Cost function of the Reserves: Demand-supply mismatches that occur in real-time are typically compensated by purchasing additional energy in the spot market. We denote by $c_s : \mathbb{R}^T \to \mathbb{R}$ the production function of the reserves so that $c_s(\mathbf{g}_s)$ is the cost incurred by the reserve generator for producing the energy sequence $\mathbf{g}_s = [g_s(1), \ldots, g_s(T)]$. We allow for the reserves to also consume excess energy, and so a negative value of $g_s(t)$ denotes an absorption of $g_s(t)$ units of energy at time t. In case it is infeasible for the reserves to absorb energy, the cost of $T-$tuples that contain negative entries are set to infinity.

3) Cost function of the EVs: For any EV i, we mean by the term "storage sequence of the EV i on day l" a $T-$length sequence that specifies the energy stored in EV i at each time of the day. Note that the storage sequence of an EV uniquely specifies how much energy must be injected or consumed from the EV at each time of the day — a decision that, as we will see shortly, the ISO must make for every EV for optimal operation of the grid. Every EV i has associated with it a cost function $c_i^{EV} : \{1, \ldots, T\} \times \{1, \ldots, T\} \times \mathbb{R}^T \to \mathbb{R}$ that specifies the cost incurred by the EV on any day l as a function of (i) its deadline $\delta_i(l)$ on that day, (ii) the actual time $\widehat{\delta}_i(l)$ at which it disconnects from the grid on that day, and (iii) its storage sequence $\mathbf{h}_{i,l}$ on that day, and is defined as

$$\begin{aligned} c_i^{EV}(\delta_i(l), \widehat{\delta}_i(l), \mathbf{h}_{i,l}) = -v_i\big(h_{i,l}(\widehat{\delta}_i(l))\big)\mathbb{1}_{\{\widehat{\delta}_i(l) \leq \delta_i(l)\}} \\ + J_m \mathbb{1}_{\{\widehat{\delta}_i(l) > \delta_i(l)\}}, \end{aligned} \tag{2}$$

where $J_m \in \mathbb{R}_{>0}$ denotes the cost incurred by the EV if it misses its deadline. In words, if EV i has deadline $\delta_i(l)$ but disconnects from the grid only after its deadline, then it incurs a cost J_m for missing its deadline, but on the other hand, if it disconnects from the grid before its deadline, then the cost that it incurs is the negative of the energy valuation that it accrues, namely, $-v_i\big(h_{i,l}(\widehat{\delta}_i(l))\big)$.

D. Storage Policy

The stochasticity of EV departure times necessitates the ISO to devise a storage policy in order to determine the storage sequence of each EV on each day. A storage policy $\boldsymbol{\pi}$ is a collection of functions $\{\pi_1, \ldots, \pi_{n_s}\}$ where the i^{th} function

$$\pi_i : \{1, \ldots, T\} \times \{1, \ldots, T\}^{n_s} \to [0, B_i]$$

specifies the energy that must be stored in EV i at each time of the day as a function of the departure profiles of the EVs on that day. Here, B_i denotes the battery capacity of EV i. More precisely, if $\widehat{\boldsymbol{\delta}}(l) \in \{1, \ldots, T\}^{n_s}$ denotes the vector of departure times of EVs on day l, then $\pi_i(t, \widehat{\boldsymbol{\delta}}(l))$ specifies the State of Charge (SoC) of EV i at time t on day l. We denote by

$$\boldsymbol{\pi}_i(\widehat{\boldsymbol{\delta}}(l)) := [\pi_i(1, \widehat{\boldsymbol{\delta}}(l)), \ldots, \pi_i(T, \widehat{\boldsymbol{\delta}}(l))]$$

the storage sequence of EV i on day l.

For a storage policy $\boldsymbol{\pi}$ to be implementable by the ISO, it must not charge or discharge any EV after it disconnects from the grid. That is, the policy $\boldsymbol{\pi}$ must satisfy the condition that for every $i \in \{1, \ldots, n_s\}$ and every $\widehat{\boldsymbol{\delta}}(l) \in \{1, \ldots, T\}^{n_s}$,

$$\pi_i(t, \widehat{\boldsymbol{\delta}}(l)) = \pi_i(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l)) \text{ for all } t \geq \widehat{\delta}_i(l).$$

I.e., the State of Charge (SoC) of the EV at any time t after its deadline is the same as the SoC of the EV at its deadline. We denote by Π the set of all implementable storage policies.

It may be desirable to constrain the number of charge-discharge cycles that an EV is subjected to while connected to the grid. This can be ensured by restricting Π to contain only those storage policies that adhere to this constraint.

E. The Independent System Operator's Objective

Prior to each day, the ISO runs a day-ahead market in which it must decide the energy dispatch sequence of the generator and the storage policy of the EVs for that day. As mentioned before, the deadlines of the EVs for any given day realize only after the day-ahead market for that day closes, and so the only information on which the ISO can base its day-ahead market decisions are the deadline distributions that the EVs bid in the day-ahead market. Suppose for a moment that the EVs are not strategic and that they bid the deadline distributions $\boldsymbol{\theta}$ truthfully. How should the ISO compute the energy dispatch sequence and the storage policy? Since the ISO does not know the deadline realizations in the day-ahead market, it chooses the energy dispatch sequence and the storage policy so as to minimize the expected cost of meeting the demand on the following day. To elaborate, suppose that the ISO decides the generator's energy dispatch sequence to be $\mathbf{g} = [g(1), \ldots, g(T)]$ and the storage policy to be $\boldsymbol{\pi}$ in the day-ahead market. Then,

$$\begin{aligned} g_s(t, \boldsymbol{\delta}(l), \mathbf{g}, \boldsymbol{\pi}) = d(t) - \Bigg[g(t) + \sum_{i=1}^{n_s} \bigg(\pi_i(t-1, \boldsymbol{\delta}(l)) \\ - \pi_i(t, \boldsymbol{\delta}(l))\bigg)\Bigg] \end{aligned} \tag{3}$$

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

is the real-time demand-supply mismatch at time t on day l. Hence, the ISO has to purchase the energy sequence

$$\mathbf{g}_s(\boldsymbol{\delta}(l), \mathbf{g}, \boldsymbol{\pi}) = [g_s(1, \boldsymbol{\delta}(l), \mathbf{g}, \boldsymbol{\pi}), \ldots, g_s(T, \boldsymbol{\delta}(l), \mathbf{g}, \boldsymbol{\pi})]$$

in the spot market on day l at price $c_s(\mathbf{g}_s(\boldsymbol{\delta}(l), \mathbf{g}, \boldsymbol{\pi}))$. Therefore, the total cost of satisfying the demand on day l — defined as the sum of the costs incurred by the generator, the reserves, and the EVs — is $c_g(\mathbf{g}) + c_s(\mathbf{g}_s(\boldsymbol{\delta}(l), \mathbf{g}, \boldsymbol{\pi})) - \sum_{j=1}^{n_s} v_j\big(\pi_j(\delta_j(l), \boldsymbol{\delta}(l))\big)$. Since $\boldsymbol{\delta}(l)$ is a random variable, the above cost is a random variable for any $(\mathbf{g}, \boldsymbol{\pi})$. The ISO's objective is to minimize the total expected cost of meeting the demand, and therefore must choose the energy dispatch sequence $\mathbf{g}^*$ and the storage policy $\boldsymbol{\pi}^*$ as a solution to the stochastic program

$$\underset{\mathbf{g}\in\mathbb{R}^T, \boldsymbol{\pi}\in\Pi}{\text{Minimize}} \ \mathbb{E}_{\boldsymbol{\delta}\sim\mathbb{P}_{\boldsymbol{\theta}}}\Big[c_g(\mathbf{g}) + c_s\big(\mathbf{g}_s(\boldsymbol{\delta}, \mathbf{g}, \boldsymbol{\pi})\big) - \sum_{j=1}^{n_s} v_j\big(\pi_j(\delta_j, \boldsymbol{\delta})\big)\Big]. \quad (4)$$

The stochastic program (4) is a two-stage stochastic program [18, Section 2.1] where $\mathbf{g}$ is the first-stage decision (also referred to as the here-and-now decision) and $\boldsymbol{\pi}$ specifies the optimal second-stage decision for each realization of $\boldsymbol{\delta}$ (also referred to as wait-and-see decision). While computational aspects are beyond the scope of this paper, suffice to say that under routine assumptions of convexity, (4) can be solved efficiently.

To make the notation compact, define

$$\beta(\boldsymbol{\delta}, \mathbf{g}, \boldsymbol{\pi}) := c_g(\mathbf{g}) + c_s(\mathbf{g}_s(\boldsymbol{\delta}, \mathbf{g}, \boldsymbol{\pi})) - \sum_{j=1}^{n_s} v_j(\pi_j(\delta_j, \boldsymbol{\delta})). \quad (5)$$

The ISO's problem (4) can then be expressed compactly as

$$\underset{\mathbf{g}\in\mathbb{R}^T, \boldsymbol{\pi}\in\Pi}{\text{Minimize}} \ \mathbb{E}_{\boldsymbol{\delta}\sim\mathbb{P}_{\boldsymbol{\theta}}}\Big[\beta(\boldsymbol{\delta}, \mathbf{g}, \boldsymbol{\pi})\Big]. \quad (6)$$

We define three functions based on (6). First, we define $\mathbf{g}^* : \Theta^{n_s} \to \mathbb{R}^T$ as a function that maps the EV parameters to an energy dispatch sequence that solves (6). Specifically, for $\psi \in \Theta^{n_s}$, $\mathbf{g}^*(\boldsymbol{\psi})$ denotes an optimal energy dispatch sequence that solves (6) if the EV parameters are $\boldsymbol{\psi}$. Similarly, we define $\boldsymbol{\pi}^* : \Theta^{n_s} \to \Pi$ as the function that maps the EVs' parameters to an optimal storage policy that solves (6). Finally, we define $q^* : \Theta^{n_s} \to \mathbb{R}$ as the function that maps the EVs' parameters to the optimal average cost so that $q^*(\boldsymbol{\psi})$ denotes the optimal value of (6) if the EV parameters are $\boldsymbol{\psi}$. We will assume throughout that the cost functions c_g and c_s are such that for some $\overline{q} < \infty$ and for all $\boldsymbol{\psi} \in \Theta^{n_s}$,

$$q^*(\boldsymbol{\psi}) \leq \overline{q}. \quad (7)$$

Hence, the ISO's objective in the day-ahead market is to compute the functions $\mathbf{g}^*$ and $\boldsymbol{\pi}^*$ at the point $\boldsymbol{\theta}$. However, if the EVs are strategic, then they may not bid their deadline distributions truthfully and so the ISO has the additional task of eliciting $\boldsymbol{\theta}$ truthfully. Our main objective is to devise a mechanism that enables the ISO to do this.

F. The EV user's Objective

On each day l, each EV i receives a payment $p_i(l)$ from the ISO in return for leasing its battery to the grid. Denoting by $\mathbf{h}_{i,l}$ the storage sequence of EV i on day l, its utility on that day is defined as

$$u_i(\delta_i(l), \widehat{\delta}_i(l), \mathbf{h}_{i,l}) := p_i(l) - c_i^{EV}(\delta_i(l), \widehat{\delta}_i(l), \mathbf{h}_{i,l}).$$

Each EV i's objective is to maximize its long-term average utility defined as $\liminf_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} u_i(\delta_i(l), \widehat{\delta}_i(l), \mathbf{h}_{i,l})$.

G. The Market Process

There are two impediments to the ISO operating the grid at the optimal cost $q^*(\boldsymbol{\theta})$. The first is the ISO's nescience of the parameter vector $\boldsymbol{\theta}$, which renders it incapable of computing the optimal decisions $g^*(\boldsymbol{\theta})$ and $\boldsymbol{\pi}^*(\boldsymbol{\theta})$ in the day-ahead market. As mentioned before, an EV's deadline distribution is its private knowledge and unknown to the ISO. Consequently, the ISO requests each EV to report its deadline distribution in the day-ahead market so that it can compute the optimal energy dispatch sequence and storage policy. However, since the objective of any EV is only to maximize its own utility, it may misreport its deadline distribution if there is a possibility for it to extract a higher utility by doing so than by bidding truthfully. Consequently, we denote by $\widehat{\theta}_i \in \Theta$ the parameter reported by EV i in the day-ahead market, which may or may not be equal to θ_i.

Based on the reported parameters $\widehat{\boldsymbol{\theta}} := [\widehat{\theta}_1, \ldots, \widehat{\theta}_{n_s}]$, the ISO computes the energy dispatch as $\mathbf{g}^*(\widehat{\boldsymbol{\theta}})$ and the storage policy as $\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})$. Since the deadline distributions of the EVs are assumed to remain the same on all days, it suffices for the EVs to report their parameters just once, and for the ISO to run the day-ahead market just once, namely, before day 1, to compute the above decisions. Once the ISO computes these quantities, it reuses them on all days.

In the day-ahead market corresponding to any day l, the ISO schedules the generator to produce the energy sequence $\mathbf{g}^*(\widehat{\boldsymbol{\theta}})$ on day l and decides on the storage policy $\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})$ for day l. After the day-ahead market closes, the EVs observe their respective deadlines for that day. The ISO requests the EVs to report their deadline realizations at the commencement of day l. Based on the reported deadlines, the ISO computes the storage sequence of each EV for that day using the policy $\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})$. Being strategic, the EVs may not bid their deadline realizations truthfully, and so we denote by $\widehat{\delta}_i(l)$ the deadline reported by EV i on day l. Having bid $\widehat{\delta}_i(l)$ as its deadline, EV i is obligated to remain connected to the grid until time $\widehat{\delta}_i(l)$ on day l. The entire chronology is illustrated in Fig. 1.

H. The Mechanism Design Problem

Having fixed $\mathbf{g}^*(\widehat{\boldsymbol{\theta}})$ and $\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})$ in the day-ahead market, the long-term average utility that EV i accrues is

$$u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^{\infty}, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^{\infty}) := \liminf_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} u_i\big(\delta_i(l), \widehat{\delta}_i(l), \boldsymbol{\pi}_i^*(\widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big), \quad (8)$$

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

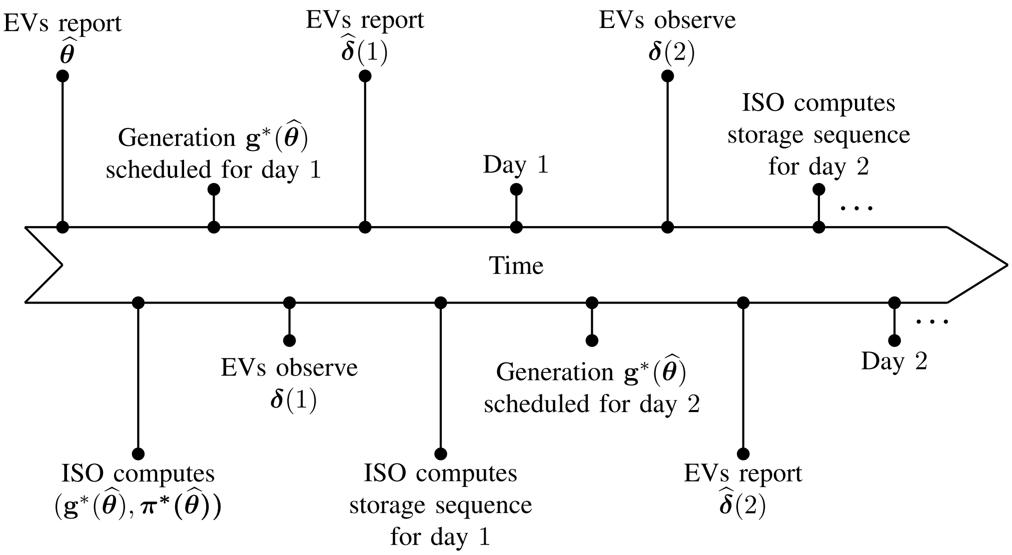

Fig. 1. The market chronology: In the day-ahead market before day 1, each EV i reports a deadline distribution to the ISO, based on which the latter computes the energy dispatch sequence and the storage policy. Then, at the commencement of any day l, $l \in \mathbb{Z}_+$, each EV i observes its deadline $\delta_i(l)$ for that day. Following this, EV i reports $\widehat{\delta}_i(l)$ as its deadline, which could potentially be adapted to $\delta_i^l, \widehat{\delta}_i^{l-1}, \boldsymbol{\delta}_{-i}^l, \widehat{\boldsymbol{\delta}}_{-i}^l, \boldsymbol{\theta}$, and $\widehat{\boldsymbol{\theta}}$. Based on $\widehat{\boldsymbol{\delta}}(l)$, the ISO computes the storage schedule for each EV for that day using the storage policy. Day l then progresses, and the process repeats on day $l+1$.

where $\boldsymbol{\pi}_i^*(\widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})$ denotes the function $\boldsymbol{\pi}_i^*(\widehat{\boldsymbol{\theta}})$ evaluated at $\widehat{\boldsymbol{\delta}}(l)$. Consequently, if there exists $(\widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^{\infty})$ such that EV i obtains a higher value for (8) by misreporting either or both θ_i and δ_i^{∞}, then it may do so. However, unless all EVs report their respective parameters truthfully in the day-ahead market and report their respective deadlines truthfully "almost all days," the ISO cannot ensure that the long-term average cost of meeting the demand approaches the optimal value $q^*(\boldsymbol{\theta})$. This brings us to the central problem that is addressed in the paper, namely, that of designing a mechanism that incentivizes EVs to report not only their deadline distributions truthfully in the day-ahead market, but also report their deadline realizations truthfully almost all days. Specifically, we aim to design a mechanism that renders truth-telling a dominant strategy so that for every EV i, its average utility $u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^{\infty}, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^{\infty})$ is maximized by setting $\widehat{\theta}_i = \theta_i$ and $\widehat{\delta}_i(l) = \delta_i(l)$ for all $l \in \mathbb{Z}_+$, regardless of what $(\widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^{\infty})$ is. The next section develops the mechanism and establishes the incentive and optimality properties guaranteed by it.

IV. Mechanism for Trading Storage Capacity of EVs with Stochastic Deadlines

In this section, we develop a mechanism that renders truth-telling an individually rational dominant strategy for every EV.

First, the ISO, which plays the role of a social planner, computes $\mathbf{g}^*(\widehat{\boldsymbol{\theta}})$ and $\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})$ in the day-ahead market as the generator's energy dispatch sequence and the storage policy. Recall that these quantities solve the stochastic program

$$\underset{\mathbf{g},\boldsymbol{\pi}}{\text{Min.}} \;\; c_g(\mathbf{g}) + \mathbb{E}_{\boldsymbol{\delta} \sim \mathbb{P}_{\widehat{\boldsymbol{\theta}}}} \Big[c_s(\mathbf{g}_s(\boldsymbol{\delta}, \mathbf{g}, \boldsymbol{\pi})) - \sum_{j=1}^{n_s} v_j(\pi_j(\delta_j, \boldsymbol{\delta})) \Big], \tag{9}$$

and note that the solutions of (6) and (9) coincide if $\widehat{\boldsymbol{\theta}} = \boldsymbol{\theta}$.

We next describe the payment rule. The payment rule consists of each EV receiving two payments on each day — a "day-ahead payment" that is determined based on the parameters reported to the ISO in the day-ahead market, and an "end-of-the-day settlement" that is determined at the end of each day based on a comparison of the the actual departure profile of the EVs with the deadline distributions reported in the day-ahead market.

A. The Day-Ahead Payment

The day-ahead payment takes the form of a VCG payment. For each $i \in \{1, \ldots, n_s\}$, define $\mathbb{P}_{\widehat{\boldsymbol{\theta}}_{-i}} := \mathbb{P}_{\widehat{\theta}_1} \times \ldots \times \mathbb{P}_{\widehat{\theta}_{i-1}} \times \mathbb{P}_{\widehat{\theta}_{i+1}} \times \ldots \times \mathbb{P}_{\widehat{\theta}_{n_s}}$ and let $q^*(\widehat{\boldsymbol{\theta}}_{-i})$ be the optimal value of the stochastic program

$$\underset{\mathbf{g},\boldsymbol{\pi}}{\text{Min.}} \;\; c_g(\mathbf{g}) + \mathbb{E}_{\boldsymbol{\delta}_{-i} \sim \widehat{\boldsymbol{\theta}}_{-i}} \big[c_s(\mathbf{g}_s(\boldsymbol{\delta}_{-i}, \mathbf{g}, \boldsymbol{\pi})) - \sum_{j \neq i} v_j(\pi_j(\delta_j, \boldsymbol{\delta}_{-i})) \big].$$

Note that this is the optimization problem that the ISO would have had to solve in the day-ahead market if EV i were absent from the system. The day-ahead payment of EV i, $i \in \{1, \ldots, n_s\}$, is defined as

$$\begin{aligned} p_i^{DA}(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) := q^*(\widehat{\boldsymbol{\theta}}_{-i}) - \Big[& c_g(\mathbf{g}^*(\widehat{\boldsymbol{\theta}})) \\ & + \mathbb{E}_{\boldsymbol{\delta} \sim \widehat{\boldsymbol{\theta}}} \Big\{ c_s(\mathbf{g}_s(\boldsymbol{\delta}, \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))) \\ & - \sum_{j \neq i} v_j(\pi_j^*(\delta_j, \boldsymbol{\delta}; \widehat{\boldsymbol{\theta}})) \Big\} \Big]. \end{aligned} \tag{10}$$

B. The End-of-the-Day Settlement

Arbitrarily fix an EV i. One of the functionalities of the end-of-the-day settlement is to penalize the EV for deviations of

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

its empirically observed departure times from $\mathbb{P}_{\widehat{\theta}_i}$. To do this, on each day l and for each $t \in \{1, \dots, T\}$, the end-of-the-day settlement function constructs a window of size $r(l)$ centered at $\mathbb{P}_{\widehat{\theta}_i}(t)$ and penalizes the EV if the empirical frequency $\frac{1}{l}\sum_{l'=1}^{l} \mathbb{1}_{\{\widehat{\delta}_i(l')=t\}}$ falls outside the window. Towards this, for $i \in \{1, \dots, n_s\}$, $t \in \{1, \dots, T\}$, and $l \in \mathbb{Z}_+$, define

$$f_{i,t}(l, \widehat{\delta}_i^l, \widehat{\theta}_i) := \left[\frac{1}{l}\sum_{l'=1}^{l} \mathbb{1}_{\{\widehat{\delta}_i(l')=t\}}\right] - \mathbb{P}_{\widehat{\theta}_i}(t), \qquad (11)$$

and define the event

$$E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i) := \{\max_{t \in \{1,\dots,T\}} |f_{i,t}(l, \widehat{\delta}_i^l, \widehat{\theta}_i)| \geq r(l)\} \qquad (12)$$

where $r(l)$ is the window size on day l. The occurrence of the event $E_i(l, \widehat{\delta}_i^l, \widehat{\theta})$ indicates that the empirical frequency of the reported deadlines of EV i up to day l is "significantly different" from the deadline distribution that it had reported in the day-ahead market. If this happens, the mechanism imposes a penalty of $J_p(l)$ on EV i on day l, where $\{J_p\}$ is chosen as any nonnegative sequence satisfying

$$\lim_{l \to \infty} \frac{J_p(l)}{l} = \infty. \qquad (13)$$

Now, the window size sequence $\{r(l)\}$ must be designed carefully so as to balance two competing objectives. On the one hand, the window size must approach zero as l tends to infinity. If not, the set of sequences from which the EV can choose its real-time bids $\widehat{\delta}_i^\infty$ without incurring a penalty would be "large," thereby resulting in the violation of incentive compatibility. On the other hand, if the window size shrinks too quickly, then the empirical frequency sequence of truthful bidders will fall outside the window infinitely often. This would result in truthful bidders paying a penalty infinitely often, thereby resulting in the violation of their individual rationality. This brings us to the question of what the appropriate rate is at which the window size should decay. Condition (14) below provides an answer.

The window size sequence $\{r\}$ is chosen such that for some $L_r \in \mathbb{N}$ and some $\gamma > \frac{1}{2}$,

$$r(l) \geq \sqrt{\frac{\ln l^\gamma}{l}} \qquad (14)$$

for all $l \geq L_r$, and

$$\lim_{l \to \infty} r(l) = 0. \qquad (15)$$

To provide some intuition for the decay rate (14), note that the empirical frequency $\frac{1}{l}\sum_{l'=1}^{l} \mathbb{1}_{\{\delta_i(l')=t\}}$ of EV i's true deadlines is approximately normally distributed for large l with a standard deviation that scales as $\frac{1}{\sqrt{l}}$. Hence, scaling the window size also at the same rate would result in the probability of empirical frequencies of truthful bids falling outside the window to remain at some fixed value which does not scale with l. To avoid this, the window size must scale slower than at least $\frac{1}{\sqrt{l}}$. Lemma 1 shows that by scaling it only "slightly" slower than this rate, namely, at the rate specified by (14), truthful bidders are guaranteed to not incur a penalty.

The end-of-the-day settlement of EV i on day l is defined

$$\begin{aligned} p_i^S(l, \widehat{\boldsymbol{\theta}}, \widehat{\delta}_i^l, \widehat{\boldsymbol{\delta}}(l)) \\ := \Big[\mathbb{E}_{\boldsymbol{\delta} \sim \mathbb{P}_{\widehat{\boldsymbol{\theta}}}}\{v_i(\pi_i^*(\delta_i, \boldsymbol{\delta}; \widehat{\boldsymbol{\theta}}))\} - v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big)\Big] \\ - J_p(l)\mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}}. \end{aligned} \qquad (16)$$

The total payment p_i received by EV i on day l is the sum of its day-ahead payment and its end-of-the-day settlement:

$$p_i(l, \widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\delta}_i^l, \widehat{\boldsymbol{\delta}}(l)) = p_i^{DA}(\widehat{\theta}_i, \widehat{\theta}_{-i}) + p_i^S(l, \widehat{\boldsymbol{\theta}}, \widehat{\delta}_i^l, \widehat{\boldsymbol{\delta}}(l)). \qquad (17)$$

The following theorem establishes the incentive and efficiency properties of the mechanism defined by the decision rule (9) and the payment rule (17), and is the central result of the paper.

Theorem 1. *Suppose that the ISO determines $\mathbf{g}^*(\widehat{\boldsymbol{\theta}})$ and $\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})$ as a solution to (9) and determines the payments according to (17). Then, for J_m sufficiently large, the following hold.*

1) *For every $i \in \{1, \dots, n_s\}$ and every $\theta_i \in \Theta$, there exists $\mathcal{E}_i \subset \{1, \dots, T\}^\infty$ with $\mathbb{P}_{\theta_i}^\infty(\mathcal{E}_i) = 0$ such that for every $\delta_i^\infty \notin \mathcal{E}_i$,*

$$u_i^{\text{avg}}(\theta_i, \delta_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \geq u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \qquad (18)$$

for every $(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\delta}}^\infty)$.
I.e., for every EV i, truth-telling is $\mathbb{P}_{\theta_i}^\infty-$ almost surely a dominant strategy.

2) *Let $i \in \{1, \dots, n_s\}$ and suppose that θ_i is such that for all $\widehat{\theta}_i \neq \theta_i$,*

$$q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \neq q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) \qquad (19)$$

for some $\widehat{\boldsymbol{\theta}}_{-i} \in \Theta^{n_s - 1}$. If for some $\widehat{\theta}_i \in \Theta$, there exists $\mathcal{E}_i \subset \{1, \dots, T\}^\infty$ with $\mathbb{P}_{\theta_i}^\infty(\mathcal{E}_i) = 0$ such that for every $\delta_i^\infty \notin \mathcal{E}_i$, $\widehat{\boldsymbol{\theta}}_{-i}$, and $\widehat{\boldsymbol{\delta}}_{-i}^\infty$, there exists $\widehat{\delta}_i^\infty$ such that

$$u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) = u_i^{\text{avg}}(\theta_i, \delta_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty), \qquad (20)$$

then,

$$\widehat{\theta}_i = \theta_i, \qquad (21)$$

and

$$\lim_{L \to \infty} \frac{1}{L}\sum_{l=1}^{L} \mathbb{1}_{\{\widehat{\delta}_i(l) \neq \delta_i(l)\}} = 0. \qquad (22)$$

I.e., for every $i \in \{1, \dots, n_s\}$ such that (19) holds, truth-telling in the day-ahead market and truth-telling on almost all days is $\mathbb{P}_{\theta_i}^\infty-$ almost surely the unique dominant strategy for EV i.

3) *For every $i \in \{1, \dots, n_s\}$ and every $\theta_i \in \Theta$, there exists $\mathcal{E}_i \subset \{1, \dots, T\}^\infty$ with $\mathbb{P}_{\theta_i}^\infty(\mathcal{E}_i) = 0$ such that for all $\delta_i^\infty \notin \mathcal{E}_i$,*

$$u_i^{\text{avg}}(\theta_i, \delta_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \geq 0 \qquad (23)$$

for all $\widehat{\boldsymbol{\theta}}_{-i}$ and $\widehat{\boldsymbol{\delta}}_{-i}^\infty$.

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

I.e., for every EV i, truth-telling is $\mathbb{P}_{\theta_i}^{\infty}-$ almost surely individually rational.

4) *If (19) and (20) hold for every $i \in \{1, \ldots, n_s\}$, then*

$$\limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \beta\big(\widehat{\boldsymbol{\delta}}(l), \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})\big) = q^*(\boldsymbol{\theta}) \quad (24)$$

$\mathbb{P}_{\boldsymbol{\theta}}^{\infty}-$ almost surely.
I.e., if every EV employs a dominant strategy, then the time-averaged cost at which the ISO satisfies the demand is equal to the optimal average cost at which it satisfies the demand if all EVs are truthful.

Proof. Arbitrarily fix $i \in \{1, \ldots, n_s\}$ and $\theta_i \in \Theta$. We begin the proof with four lemmas. The first lemma assures that honest bidding almost surely yields zero penalty.

Lemma 1. *There exists $\mathcal{E}_i \subset \{1, \ldots, T\}^\infty$ with $\mathbb{P}_{\theta_i}^{\infty}(\mathcal{E}_i) = 0$ such that for all $\delta_i^\infty \notin \mathcal{E}_i$,*

$$\lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} J_p(l) \mathbb{1}_{\{E_i(l, \delta_i^l, \theta_i)\}} = 0. \quad (25)$$

Proof. See Appendix A. □

The second lemma establishes the monotonicity of the optimal cost function q^* in a certain sense.

Lemma 2. *Let $\lambda_i, \widetilde{\lambda}_i \in \Theta$ be any two parameters such that $F_{\widetilde{\lambda}_i}(t) \geq F_{\lambda_i}(t)$ for all $t \in \{1, \ldots, T\}$. Then,*

$$q^*(\widetilde{\lambda}_i, \widehat{\boldsymbol{\theta}}_{-i}) \geq q^*(\lambda_i, \widehat{\boldsymbol{\theta}}_{-i})$$

for all $\widehat{\boldsymbol{\theta}}_{-i} \in \Theta^{n_s - 1}$.

Proof. The proof is relatively straightforward and has been omitted in the interest of space. See [19] for details. □

Lemma 3. *Suppose that $\widehat{\theta}_i$ is such that*

$$\alpha(\theta_i, \widehat{\theta}_i) := \sup_{t \in \{1, \ldots, T\}} \{F_{\theta_i}(t) - F_{\widehat{\theta}_i}(t)\} > 0. \quad (26)$$

Then,

1) *for some finite $K \geq 0$,*

$$q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \leq K \alpha(\theta_i, \widehat{\theta}_i) \quad (27)$$

for all $\widehat{\boldsymbol{\theta}}_{-i} \in \Theta^{n_s - 1}$.

2) *There exists $\mathcal{E}_i \subset \{1, \ldots, T\}^\infty$ with $\mathbb{P}_{\theta_i}^{\infty}(\mathcal{E}_i) = 0$ such that for all $\delta_i^\infty \notin \mathcal{E}_i$,*

$$\lim_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \mathbb{1}_{\{\widehat{\delta}_i(l) > \delta_i(l)\}} \geq \alpha(\theta_i, \widehat{\theta}_i) \quad (28)$$

whenever $\widehat{\delta}_i^\infty$ is such that $\sum_{l=1}^{\infty} \mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}} < \infty$.

Proof. See Appendix B. □

Lemma 4. *If $\widehat{\delta}_i^\infty$ is such that $\sum_{l=1}^{\infty} \mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}} = \infty$, then,*

$$\limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} J_p(l) \mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}} = \infty. \quad (29)$$

Proof. See Appendix C. □

We are now ready to prove the theorem. We first have

$$\begin{aligned} &u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \\ &\quad = p_i^{DA}(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \\ &\qquad + \liminf_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \Big[p_i^S(l, \widehat{\boldsymbol{\theta}}, \widehat{\delta}_i^l, \widehat{\boldsymbol{\delta}}(l)) \\ &\qquad\quad - c_i^{EV}\big(\delta_i(l), \widehat{\delta}_i(l), \boldsymbol{\pi}_i^*(\widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big)\Big]. \end{aligned}$$

Substituting (2), (10) and (16) in the above equality and carrying out some algebra yields

$$\begin{aligned} &u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \\ &\quad = \big[q^*(\widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\boldsymbol{\theta}})\big] \\ &\qquad - \limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \Big[J_p(l) \mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}} \\ &\qquad\quad + J_m \mathbb{1}_{\{\widehat{\delta}_i(l) > \delta_i(l)\}} \Big] \\ &\qquad - \limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \Big[v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big) \\ &\qquad\quad - v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big) \mathbb{1}_{\{\widehat{\delta}_i(l) \leq \delta_i(l)\}} \Big]. \end{aligned} \quad (30)$$

Using the above expression, applying Lemma 1, and carrying out some algebra implies the existence of $\mathcal{E}_i \subset \{1, \ldots, T\}^\infty$ with $\mathbb{P}_{\theta_i}^{\infty}(\mathcal{E}_i) = 0$ such that for all $\delta_i^\infty \notin \mathcal{E}_i$,

$$\begin{aligned} &u_i^{\text{avg}}(\theta_i, \delta_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \\ &\quad - u_i^{\text{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \\ &\quad = \big[q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i})\big] \\ &\qquad + \limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \Big[J_p(l) \mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}} \Big] \\ &\qquad + \limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} \Big[v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big) \\ &\qquad\quad - v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big) \mathbb{1}_{\{\widehat{\delta}_i(l) \leq \delta_i(l)\}} \Big] \\ &\qquad + \limsup_{L\to\infty} \frac{1}{L} \sum_{l=1}^{L} J_m \mathbb{1}_{\{\widehat{\delta}_i(l) > \delta_i(l)\}}. \end{aligned} \quad (31)$$

We show that the above random variable is nonnegative, thereby establishing (18). We do this by considering three cases and showing that (31) is nonnegative in all the three cases.

First consider the case when $\widehat{\delta}_i^\infty$ is such that $\sum_{l=1}^{\infty} \mathbb{1}_{\{E_i(l, \widehat{\delta}_i^l, \widehat{\theta}_i)\}} = \infty$. Since $\big|q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i})\big| < \infty$ from (7) and all other terms in the RHS of (31) are nonnegative, using Lemma 4 in (31) implies the nonnegativity of (31).

Next consider the case when $\widehat{\theta}_i$ is such that $F_{\theta_i}(t) \leq F_{\widehat{\theta}_i}(t)$ for all $t \in \{1, \ldots, T\}$. It then follows from Lemma 2 that $q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \geq q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i})$, and so the first term in the RHS of (31) is nonnegative. Since every other term in the RHS of (31) is nonnegative, (31) is nonnegative.

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

We are finally left with the case when $\widehat{\delta}_i^\infty$ is such that $\sum_{l=1}^{\infty} \mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}} < \infty$ and $\widehat{\theta}_i$ is such that $F_{\theta_i}(t) > F_{\widehat{\theta}_i}(t)$ for some $t \in \{1,\dots,T\}$. Lemma 3 applies, and so combining (27) and (28) with (31) implies that $\mathbb{P}_{\theta_i}^\infty-$ almost surely,

$$u_i^{\mathrm{avg}}(\theta_i, \delta_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) - u_i^{\mathrm{avg}}(\widehat{\theta}_i, \widehat{\delta}_i^\infty, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^\infty) \geq (J_m - K)\alpha(\theta_i, \widehat{\theta}_i).$$

It follows that for $J_m \geq K$, the LHS is nonnegative, thereby completing the proof of (18).

We next prove the second statement of the theorem. Suppose that (20) holds. Using (30) to expand both sides of (20), invoking Lemma 1, and simplifying the result implies that for every $\delta_i^\infty \notin \mathcal{E}_i$, $\widehat{\boldsymbol{\theta}}_{-i}$, $\widehat{\boldsymbol{\delta}}^\infty$, there exists $\widehat{\delta}_i^\infty$ such that

$$\begin{aligned} q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) &- q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \\ &= \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} J_p(l)\mathbb{1}_{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)} \\ &\quad + \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\Big[v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \boldsymbol{\theta})\big) \\ &\qquad - v_i\big(\pi_i^*(\widehat{\delta}_i(l), \widehat{\boldsymbol{\delta}}(l); \widehat{\boldsymbol{\theta}})\big)\mathbb{1}_{\{\widehat{\delta}_i(l)\leq \delta_i(l)\}}\Big] \\ &\qquad\quad + J_m \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{\widehat{\delta}_i(l) > \delta_i(l)\}}. \end{aligned} \tag{32}$$

Since the RHS of the above equality is always nonnegative, we have that $q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \geq 0$ for every $\widehat{\boldsymbol{\theta}}_{-i}$. We show next that this inequality must hold with equality for all $\widehat{\boldsymbol{\theta}}_{-i}$.

Suppose for contradiction that $q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) > 0$ for some $\widehat{\boldsymbol{\theta}}_{-i}$. Lemma 2 implies that $\sup_{t\in\{1,\dots,T\}}\{F_{\theta_i}(t) - F_{\widehat{\theta}_i}(t)\} = \alpha(\theta_i, \widehat{\theta}_i) > 0$, and so Lemma 3 applies. Hence,

$$q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \leq K\alpha(\theta_i, \widehat{\theta}_i). \tag{33}$$

Since the LHS of (32) is finite following (7), the RHS must also be finite. This implies in particular that the first term of the RHS is finite, and so, it follows from Lemma 4 that $\sum_{l=1}^{\infty} \mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}} < \infty$. Consequently, (28) holds, and substituting it in (32) implies that $\mathbb{P}_{\theta_i}^\infty-$ almost surely,

$$q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \geq J_m\alpha(\theta_i, \widehat{\theta}_i). \tag{34}$$

Combining (33) and (34) yields a contradiction for $J_m > K$. Hence, if (20) holds, then,

$$q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i}) = q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) \tag{35}$$

for all $\widehat{\boldsymbol{\theta}}_{-i}$.

Now, since (19) holds, combining it with (35) immediately establishes (21), i.e., that $\widehat{\theta}_i = \theta_i$. Consequently, the RHS of (32) is zero, and since every term in the RHS of (32) is nonnegative, it follows that every term is zero. Hence,

$$\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} J_p(l)\mathbb{1}_{E_i(l,\widehat{\delta}_i^l,\theta_i)} = 0 \tag{36}$$

and

$$\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{\widehat{\delta}_i(l) > \delta_i(l)\}} = 0 \tag{37}$$

$\mathbb{P}_{\theta_i}^\infty-$ almost surely.

It follows from (13) that

$$\limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} J_p(l)\mathbb{1}_{E_i(l;\widehat{\delta}_i^l,\theta_i)} \geq \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{E_i(l;\widehat{\delta}_i^l,\theta_i)},$$

which when combined with (36) implies

$$\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{E_i(l;\widehat{\delta}_i^l,\theta_i)} = 0.$$

This in turn implies, using (12), that for every $t \in \{1,\dots,T\}$,

$$\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)|\geq r(l)\}} = 0. \tag{38}$$

Since $|f_{i,t}(l, \widehat{\delta}_i^l, \theta_i)| \leq 1$, it follows that for all $t \in \{1,\dots,T\}$ and all $L \in \mathbb{N}$,

$$\sum_{l=1}^{L}|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)| \leq \sum_{l=1}^{L}\mathbb{1}_{\{|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)|\geq r(l)\}} + \sum_{l=1}^{L} r(l)\mathbb{1}_{\{|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)|< r(l)\}}.$$

Dividing this inequality by L and taking the limit as $L \to \infty$ implies that for all $t \in \{1,\dots,T\}$,

$$\begin{aligned} \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}&|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)| \\ &\leq \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)|\geq r(l)\}} \\ &\quad + \limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} r(l)\mathbb{1}_{\{|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)|< r(l)\}}. \end{aligned}$$

We have using (15) that the last term of the RHS of the above inequality equals zero, and using (38) that its first term is zero. Hence, for all $t \in \{1,\dots,T\}$, we have $\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}|f_{i,t}(l,\widehat{\delta}_i^l,\theta_i)| = 0$. This implies using (11) that for all $t \in \{1,\dots,T\}$,

$$\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\left[\frac{1}{l}\sum_{l'=1}^{l}\mathbb{1}_{\{\widehat{\delta}_i(l')=t\}}\right] = \mathbb{P}_{\theta_i}(t).$$

Multiplying this equality by t, summing both sides over t, and simplifying yields

$$\lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\left[\frac{1}{l}\sum_{l'=1}^{l}\widehat{\delta}_i(l')\right] = \mu(\theta_i), \tag{39}$$

where $\mu(\theta_i) := \sum_{t=1}^{T} t\mathbb{P}_{\theta_i}(t)$ is the expected value of the distribution corresponding to the parameter θ_i. I.e., the empirical mean of the reported deadlines converges to the expected value of the true deadline in a Cesàro sense.

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

Suppose for contradiction that for some $\epsilon > 0$,

$$\limsup_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L} \mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}} = \epsilon. \tag{40}$$

Note that

$$\begin{aligned}\limsup_{L\to\infty}&\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\right]\\ &= \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)=\delta_i(l)\}}\right]\\ &\quad + \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}}\right]\\ &\quad + \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)>\delta_i(l)\}}\right]. \end{aligned}\tag{41}$$

Now,

$$\begin{aligned}\limsup_{L\to\infty}&\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}}\right]\\ &< \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}}\right] + \epsilon\\ &= \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\left(\widehat{\delta}_i(l)+1\right)\mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}}\right]\\ &\le \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\delta_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}}\right],\end{aligned}$$

where the equality follows from (40). It follows from (37) that the last term in the RHS of (41) is $\mathbb{P}_{\theta_i}^{\infty}-$ almost surely zero, and so substituting this and the above inequality in (41) implies that $\mathbb{P}_{\theta_i}^{\infty}-$ almost surely,

$$\limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l)\right] < \limsup_{L\to\infty}\left[\frac{1}{L}\sum_{l=1}^{L}\delta_i(l)\mathbb{1}_{\{\widehat{\delta}_i(l)\le\delta_i(l)\}}\right].$$

It follows from (37) and SLLN that the RHS of the above inequality $\mathbb{P}_{\theta_i}^{\infty}-$ almost surely equals $\mu(\theta_i)$, and so, $\limsup_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\widehat{\delta}_i(l) < \mu(\theta_i)$. Combining this inequality with (39) yields a contradiction. Hence, $\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{\widehat{\delta}_i(l)<\delta_i(l)\}} = 0$, and combining this equality with (37) establishes (22).

We now prove the third part of the theorem. Using (30), we have

$$\begin{aligned}u_i^{\text{avg}}(\theta_i, \delta_i^{\infty}, \widehat{\boldsymbol{\theta}}_{-i}, \widehat{\boldsymbol{\delta}}_{-i}^{\infty}) &= \left[q^*(\widehat{\boldsymbol{\theta}}_{-i}) - q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i})\right]\\ &\quad - \limsup_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\left[J_p(l)\mathbb{1}_{\{E_i(l,\delta_i^l,\theta_i)\}}\right].\end{aligned}$$

Substituting Lemma 1 in the above equality and noting that $[q^*(\widehat{\boldsymbol{\theta}}_{-i}) - q^*(\theta_i, \widehat{\boldsymbol{\theta}}_{-i})] \ge 0$ establishes (23).

Finally, (24) follows by substituting (21) in the LHS of (24) and using (22) and SLLN to simplify the result. □

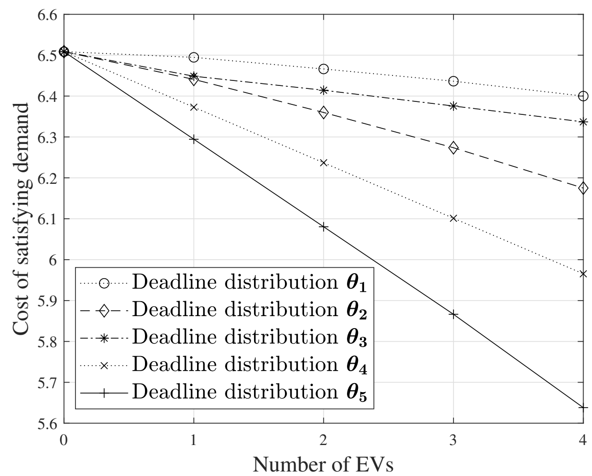

Fig. 2. The total expected cost of satisfying the demand is plotted as a function of varying levels of EV penetration for five different profiles of EV deadline distributions. The solid curve corresponds to the scenario where all EVs can lease their battery to the grid for the entire day, and denotes the minimum possible cost at which the ISO can satisfy the demand.

V. Numerical Results

In our simulations, we have divided a day into a total of five time intervals. The simulation parameters are specified in Table I. In order to obtain conservative estimates of the cost reduction, we have assumed all EVs to have a capacity of 10kWh, which is an order of magnitude lesser than the battery capacity of many EVs that are commercially available today. With the view of rendering the stochastic program (9) have modest complexity (see [19] for details), we have quantized the energy levels to which the EVs can be charged to $\{0, 10\text{kWh}\}$. We have also quantized the generator's energy output so that at any time, the generator can only produce an integer multiple of 10 kWh.

The production functions used in the simulations are

$$c_g(\mathbf{g}) = \sum_{t=1}^{5} c^g(t)g(t)$$

and

$$c_s(\mathbf{g}_s) = \sum_{t=1}^{5}\left[c^s(t)g_s(t)\mathbb{1}_{\{g_s(t)\ge 0\}} + c^s(t)g_s^2(t)\mathbb{1}_{\{g_s(t)<0\}}\right]$$

where the numerical values for $[c^g(1) \ \ldots \ c^g(5)]$ and $[c^s(1) \ \ldots \ c^s(5)]$ are specified in Table I. The function c_s can be thought of as indicating that it is in some sense easier for the reserves to produce energy in real time than to consume it. We have assumed the energy valuation functions of all EVs to be the identity function so that $v_i(h) = h$ for all $h \in \mathbb{R}$.

Fig. 2 plots the total cost $q^*(\boldsymbol{\theta})$ at which the ISO satisfies the demand for five different deadline distribution profiles of the EVs. The distribution θ_4 corresponds to later deadlines than the distribution θ_3 in that $F_{\theta_4}(t) \le F_{\theta_3}(t)$ for all t. Similarly, $F_{\theta_4}(t) \le F_{\theta_2}(t) \le F_{\theta_1}(t)$ for all t. Hence, in addition to showing the reduction in costs due to increased EV penetration, Fig. 2 also illustrates the value that later EV deadlines provide in reducing the operating cost.

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

TABLE I
SIMULATION PARAMETERS

Variable	Value	Units
$\mathbf{d}$	[36.7387 38.5138 56.6975 73.9188 57.6061]	kWh
c^g	[12.4198 18.8367 19.1754 31.0088 33.3978]	\$/MWh
c^s	[27.8936 28.2861 29.3702 30.5788 34.3765]	\$/MWh
θ_A	[0.2000 0.2000 0.2000 0.2000 0.2000]	
θ_B	[0.0770 0.2442 0.0783 0.0716 0.5290]	
θ_C	[0.0378 0.2430 0.1449 0.5683 0.0059]	
θ_D	[0.0212 0.0462 0.1019 0.2061 0.6245]	
θ_E	[0 0 0 0 1]	
$\boldsymbol{\theta}_1$	$[\theta_A\ \theta_A\ \theta_A\ \theta_A]$	
$\boldsymbol{\theta}_2$	$[\theta_B\ \theta_B\ \theta_B\ \theta_B]$	
$\boldsymbol{\theta}_3$	$[\theta_C\ \theta_C\ \theta_C\ \theta_C]$	
$\boldsymbol{\theta}_4$	$[\theta_D\ \theta_D\ \theta_D\ \theta_D]$	
$\boldsymbol{\theta}_5$	$[\theta_E\ \theta_E\ \theta_E\ \theta_E]$	

Finally and most importantly, these cost reductions can be attained only if the EVs bid their deadline distributions and deadline realizations truthfully, which in turn is guaranteed only in the presence of a mechanism that renders truth-telling a dominant strategy both in the day-ahead market and in real time. As illustrated in the example in Section I, in the absence of such a mechanism, strategic behavior of EVs could result in total costs in Fig. 2 that are in excess of 6.5 for all EV penetration levels and deadline distributions, implying that the ISO would be better off not utilizing the EVs at all for storage. The magnitude of the cost excess depends on the specifics of each EV's bidding strategy, which in turn could be unpredictable in the absence of a unique dominant strategy.

VI. EXTENSIONS AND CONCLUSION

We have considered the problem of integrating a fleet of strategic EVs with random deadlines into the grid and utilizing them for energy storage. Without appropriate incentive structures, EV-power grid integration could potentially be counterproductive to the cost- and energy-efficient operation of the grid. This fundamentally arises because of two phenomena operating in tandem — the randomness of EV usage patterns and the possibility of their strategic behavior. We have shown how this problem can be addressed by means of a carefully-designed energy storage market. Specifically, we have designed a mechanism for energy storage markets that guarantees certain incentive and optimality properties. The mechanism allows the ISO to achieve efficient EV-power grid integration and satisfy the demand at minimum possible cost.

The results of the paper can potentially be extended along several important directions. We have assumed the deadline distributions, the energy valuation functions, and the demand sequence to remain the same on all days. An immediate extension is to address the scenario where these quantities could be different on different days. Denote by $\{\boldsymbol{\theta}[1], \boldsymbol{\theta}[2], \ldots\}$ the sequence of EV parameters on different days and by $\{\mathbf{d}_1, \mathbf{d}_2, \ldots\}$ the demand sequence on different days. If the number of distinct elements in $\{\boldsymbol{\theta}[1], \boldsymbol{\theta}[2], \ldots\}$ and $\{\mathbf{d}_1, \mathbf{d}_2, \ldots\}$ are finite, then the above results can be readily extended to this more general setting. Specifically, by requiring the EVs to report their parameters in the day-ahead market on all days, categorizing each day into one of a finite number of "bins" such that the reported EV parameters and the demand sequence remain the same on all days belonging to a bin, and instantiating in parallel the mechanism presented in the paper — one for each bin — we obtain a mechanism that has the desired incentive and optimality guarantees.

We have also assumed that the deadlines of all EVs are independent random variables. Relaxing this assumption and developing an analogous mechanism for the case where the EV deadlines could be correlated is an important generalization. Extending the results to the context of multi-bus power systems is another important generalization.

REFERENCES

[1] P. Barter. Cars are parked 95% of the time. Let's check. Accessed: 01-26-2020. [Online]. Available: https://www.reinventingparking.org/2013/02/cars-are-parked-95-of-time-lets-check.html

[2] Sales of Electricity to Ultimate Customers by End-Use Sector. Accessed: 10-10-2020. [Online]. Available: https://www.eia.gov/electricity/annual/html/epa_02_08.html

[3] State motor-vehicle registrations - 2017. Accessed: 10-10-2020. [Online]. Available: https://www.fhwa.dot.gov/policyinformation/statistics/2017/mv1.cfm

[4] W. Kempton and J. Tomić, "Vehicle-to-grid power fundamentals: Calculating capacity and net revenue," *Journal of Power Sources*, vol. 144, no. 1, pp. 268 – 279, 2005. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378775305000352

[5] C. Guille and G. Gross, "A conceptual framework for the vehicle-to-grid (v2g) implementation," *Energy Policy*, vol. 37, no. 11, pp. 4379 – 4390, 2009. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0301421509003978

[6] J. de Hoog, T. Alpcan, M. Brazil, D. A. Thomas, and I. Mareels, "A market mechanism for electric vehicle charging under network constraints," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 827–836, 2016.

[7] A. Moradipari and M. Alizadeh, "Pricing and routing mechanisms for differentiated services in an electric vehicle public charging station network," *IEEE Transactions on Smart Grid*, vol. 11, no. 2, pp. 1489–1499, 2020.

[8] V. Robu, E. H. Gerding, S. Stein, D. C. Parkes, A. Rogers, and N. R. Jennings, "An online mechanism for multi-unit demand and its application to plug-in hybrid electric vehicle charging," *Journal of Artificial Intelligence Research*, vol. 48, pp. 175–230, 2013.

[9] S. Bhattacharya, K. Kar, J. H. Chow, and A. Gupta, "Extended second price auctions with elastic supply for pev charging in the smart grid," *IEEE Transactions on Smart Grid*, vol. 7, no. 4, pp. 2082–2093, 2016.

[10] W. Tushar, W. Saad, H. V. Poor, and D. B. Smith, "Economics of electric vehicle charging: A game theoretic approach," *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1767–1778, 2012.

[11] B. Satchidanandan and M. A. Dahleh, "A mechanism for selling battery storage service in day-ahead electricity markets," in *2021 American Control Conference (ACC)*, 2021, pp. 2895–2900.

[12] ——, "Economic dispatch for EV energy storage-integrated power systems," in *2022 14th International Conference on COMmunication Systems NETworkS (COMSNETS)*, 2022, pp. 707–715.

[13] W. Tang and R. Jain, "Market mechanisms for buying random wind," *IEEE Transactions on Sustainable Energy*, vol. 6, no. 4, pp. 1615–1623, 2015.

[14] N. Dahlin and R. Jain, "A two stage stochastic mechanism for selling random power," in *2019 American Control Conference (ACC)*, 2019, pp. 2602–2607.

[15] E. Y. Bitar, R. Rajagopal, P. P. Khargonekar, K. Poolla, and P. Varaiya, "Bringing wind energy to market," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1225–1235, 2012.

[16] Powerwall. Accessed: 02-01-2020. [Online]. Available: https://www.tesla.com/powerwall

[17] B. Mauch, J. Apt, P. M. Carvalho, and P. Jaramillo, "What day-ahead reserves are needed in electric grids with high levels of wind power?" *Environmental Research Letters*, vol. 8, no. 3, 2013.

[18] A. Ali, J. Z. Kolter, S. Diamond, and S. P. Boyd, "Disciplined convex stochastic programming: A new framework for stochastic optimization." in *UAI*, 2015, pp. 62–71.

[19] B. Satchidanandan and M. A. Dahleh, "An efficient and incentive-compatible mechanism for energy storage markets," *arXiv preprint arXiv:2012.11540*, 2020.

APPENDIX A
PROOF OF LEMMA 1

Proof. Denote by $\sigma_t^2 = \mathbb{P}_{\theta_i}(t)\big(1-\mathbb{P}_{\theta_i}(t)\big)$ the variance of the random variable $\mathbb{1}_{\{\delta_i(1)=t\}} - \mathbb{P}_{\theta_i}(t)$. Since δ_i^∞ is drawn IID from the distribution $\mathbb{P}_{\theta_i}$, it follows from the Central Limit Theorem (CLT) that for any $z \in \mathbb{R}$,

$$\lim_{l\to\infty} \sup_{z\in\mathbb{R}} \Big|\mathbb{P}\{\sqrt{l} f_{i,t}(l, \delta_i^l, \theta_i) \le z\} - Q\big(\frac{z}{\sigma_t}\big)\Big| = 0 \tag{42}$$

for all $t \in \{1, \ldots, T\}$, where Q denotes the cumulative distribution function of the standard normal distribution. Define

$$\widetilde{r}(l) := r(l)\sqrt{l}. \tag{43}$$

It follows from (42) that

$$\lim_{l\to\infty} \sup_{q\in\mathbb{N}} \Big|\mathbb{P}\{\sqrt{l} f_{i,t}(l, \delta_i^l, \theta_i) \le -\widetilde{r}(q)\} - Q\big(\frac{-\widetilde{r}(q)}{\sigma_t}\big)\Big| = 0.$$

Hence,

$$\lim_{l\to\infty} \Big|\mathbb{P}\{\sqrt{l} f_{i,t}(l, \delta_i^l, \theta_i) \le -\widetilde{r}(l)\} - Q\big(\frac{-\widetilde{r}(l)}{\sigma_t}\big)\Big| = 0,$$

which using (43) becomes

$$\lim_{l\to\infty} \Big|\mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i) \le -r(l)\} - Q\big(\frac{-\widetilde{r}(l)}{\sigma_t}\big)\Big| = 0.$$

The above equality implies that for every $\epsilon_0 > 0$, there exists $L_0 \in \mathbb{N}$ such that for all $l \ge L_0$,

$$-\epsilon_0 \le \mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i) \le -r(l)\} - Q\big(\frac{-\widetilde{r}(l)}{\sigma_t}\big) \le \epsilon_0.$$

Using the bound $Q(z) \le e^{-z^2/2}$ for $z \le 0$, the above inequality implies that for all $l \ge L_0$,

$$\mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i) \le -r(l)\} \le e^{-\widetilde{r}^2(l)/2\sigma_t^2} + \epsilon_0.$$

It follows from (14) and (43) that $\widetilde{r}(l) \ge \sqrt{\ln l^\gamma}$ for all $l \ge L_r$. Combining this with the above inequality and simplifying yields that for all $l \ge \max\{L_r, L_0\}$,

$$\mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i) \le -r(l)\} \le \frac{1}{l^{(\frac{\gamma}{2\sigma_t^2})}} + \epsilon_0,$$

and so, letting $\epsilon_0 \downarrow 0$ and noting that $2\sigma_t^2 \le \frac{1}{2}$ implies

$$\mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i)\} \le -r(l)\} = O\Big(\frac{1}{l^{2\gamma}}\Big).$$

Since $\gamma > \frac{1}{2}$, the above equality implies

$$\sum_{l=1}^{\infty} \mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i) \le -r(l)\} < \infty. \tag{44}$$

It can be shown by following the same sequence of arguments that

$$\sum_{l=1}^{\infty} \mathbb{P}\{f_{i,t}(l, \delta_i^l, \theta_i) \ge r(l)\} < \infty. \tag{45}$$

Combining (44) and (45) implies

$$\sum_{l=1}^{\infty} \mathbb{P}\{\big|f_{i,t}(l, \delta_i^l, \theta_i)\big| \ge r(l)\} < \infty.$$

Since the above inequality holds for arbitrary $t \in \{1, \ldots, T\}$, it follows that $\sum_{l=1}^{\infty} \mathbb{P}\{E_i(l, \delta_i^l, \theta_i)\} < \infty$. Invoking the Borel-Cantelli lemma yields

$$\sum_{l=1}^{\infty} \mathbb{1}_{\{E_i(l, \delta_i^l, \theta_i)\}} < \infty$$

almost surely, and (25) follows. □

APPENDIX B
PROOF OF LEMMA 3

Proof. Let ψ be that element of Θ that has the distribution function $F_\psi(t) = \max\{F_{\theta_i}(t), F_{\widehat{\theta}_i}(t)\}$. It is easy to see that

$$F_\psi(t) - F_{\widehat{\theta}_i}(t) \ge 0 \tag{46}$$

for all $t \in \{1, \ldots, T\}$, and that

$$\sup_{t\in\{1,\ldots,T\}} \{F_\psi(t) - F_{\widehat{\theta}_i}(t)\} = \alpha(\theta_i, \widehat{\theta}_i). \tag{47}$$

Note that for all $t \in \{1, \ldots, T\}$, $\mathbb{P}_\psi(t) - \mathbb{P}_{\widehat{\theta}_i}(t) = F_\psi(t) - F_\psi(t-1) - \big(F_{\widehat{\theta}_i}(t) - F_{\widehat{\theta}_i}(t-1)\big) \le \big|F_\psi(t) - F_{\widehat{\theta}_i}(t)\big| + \big|F_{\widehat{\theta}_i}(t-1) - F_\psi(t-1)\big| \le 2\alpha(\theta_i, \widehat{\theta}_i)$, where the last inequality follows from (46) and (47). Therefore,

$$\sup_{t\in\{1,\ldots,T\}} \{\mathbb{P}_\psi(t) - \mathbb{P}_{\widehat{\theta}_i}(t)\} \le 2\alpha(\theta_i, \widehat{\theta}_i). \tag{48}$$

For $t = 1, \ldots, T$, define

$$\beta_{\widehat{\boldsymbol{\theta}},i}(t; \mathbf{g}, \boldsymbol{\pi}) := \mathbb{E}_{\boldsymbol{\delta}\sim\mathbb{P}_{\widehat{\boldsymbol{\theta}}}}\big[\beta(\boldsymbol{\delta}, \mathbf{g}, \boldsymbol{\pi})\big|\delta_i = t\big] \tag{49}$$

so that $\beta_{\widehat{\boldsymbol{\theta}},i}(t; \mathbf{g}, \boldsymbol{\pi})$ denotes the conditional expected cost of satisfying the demand given that EV i disconnects from the grid at time t, the EV departure profiles is distributed according to $\mathbb{P}_{\widehat{\boldsymbol{\theta}}}$, the generator's energy dispatch sequence is $\mathbf{g}$, and the storage policy is $\boldsymbol{\pi}$. Note that the conditional expectation in (49) is well defined for every t, thanks to (1). Note also that

$$q^*(\widehat{\theta}_i, \widehat{\boldsymbol{\theta}}_{-i}) = \mathbb{E}_{\delta_i\sim\mathbb{P}_{\widehat{\theta}_i}}\big[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i; \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\big]. \tag{50}$$

Now,

$$\begin{aligned}
&\mathbb{E}_{\delta_i\sim\mathbb{P}_\psi}\big[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i; \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\big] \\
&\quad -\mathbb{E}_{\delta_i\sim\mathbb{P}_{\widehat{\theta}_i}}\big[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i; \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\big] \\
&\quad = \sum_{t=1}^{T} \big(\mathbb{P}_\psi(t) - \mathbb{P}_{\widehat{\theta}_i}(t)\big)\beta_{\widehat{\boldsymbol{\theta}},i}(t; \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}})) \\
&\le \Big[\sum_{t=1}^{T}(\mathbb{P}_\psi(t) - \mathbb{P}_{\widehat{\theta}_i}(t))^2\Big]^{\frac{1}{2}} \Big[\sum_{t=1}^{T}\beta^2_{\widehat{\boldsymbol{\theta}},i}(t; \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\Big]^{\frac{1}{2}} \\
&\qquad \le 2\sqrt{T}\alpha(\theta_i, \widehat{\theta}_i)\Big[\sum_{t=1}^{T}\beta^2_{\widehat{\boldsymbol{\theta}},i}(t; \mathbf{g}^*(\widehat{\boldsymbol{\theta}}), \boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\Big]^{\frac{1}{2}},
\end{aligned} \tag{51}$$

This article has been accepted for publication in IEEE Transactions on Smart Grid. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TSG.2022.3145689

where the last inequality follows from (48). It follows from (50) and the definitions of the functions $\mathbf{g}^*$ and $\boldsymbol{\pi}^*$ that

$$\mathbb{E}_{\delta_i\sim\mathbb{P}_\psi}\left[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i;\mathbf{g}^*([\psi,\widehat{\boldsymbol{\theta}}_{-i}]),\boldsymbol{\pi}^*([\psi,\widehat{\boldsymbol{\theta}}_{-i}]))\right] - \mathbb{E}_{\delta_i\sim\mathbb{P}_\psi}\left[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i;\mathbf{g}^*(\widehat{\boldsymbol{\theta}}),\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\right] \le 0. \quad (52)$$

Adding (51) and (52) yields

$$\mathbb{E}_{\delta_i\sim\mathbb{P}_\psi}\left[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i;\mathbf{g}^*([\psi,\widehat{\boldsymbol{\theta}}_{-i}]),\boldsymbol{\pi}^*([\psi,\widehat{\boldsymbol{\theta}}_{-i}]))\right] - \mathbb{E}_{\delta_i\sim\mathbb{P}_{\widehat{\theta}_i}}\left[\beta_{\widehat{\boldsymbol{\theta}},i}(\delta_i;\mathbf{g}^*(\widehat{\boldsymbol{\theta}}),\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\right] \le 2\sqrt{T}\alpha(\theta_i,\widehat{\theta}_i)\Big[\sum_{t=1}^{T}\beta^2_{\widehat{\boldsymbol{\theta}},i}(t;\mathbf{g}^*(\widehat{\boldsymbol{\theta}}),\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\Big]^{\frac{1}{2}},$$

which using (50) implies

$$q^*(\psi,\widehat{\boldsymbol{\theta}}_{-i}) - q^*(\widehat{\theta}_i,\widehat{\boldsymbol{\theta}}_{-i}) \le K\alpha(\theta_i,\widehat{\theta}_i) \quad (53)$$

where

$$K \coloneqq \sup_{\widehat{\boldsymbol{\theta}}\in\Theta^{n_s}} 2\sqrt{T}\Big[\sum_{t=1}^{T}\beta^2_{\widehat{\boldsymbol{\theta}},i}(t;\mathbf{g}^*(\widehat{\boldsymbol{\theta}}),\boldsymbol{\pi}^*(\widehat{\boldsymbol{\theta}}))\Big]^{\frac{1}{2}}.$$

That K is finite follows from (7).

Since $F_\psi(t) \ge F_{\theta_i}(t)$ for all $t \in \{1,\ldots,T\}$, it follows from Lemma 2 that $q^*(\theta_i,\widehat{\boldsymbol{\theta}}_{-i}) - q^*(\psi,\widehat{\boldsymbol{\theta}}_{-i}) \le 0$. Adding this inequality with (53) establishes (27).

We now turn attention to the second part of the lemma. Let $\widehat{\theta}_i$ be such that (26) holds and denote by t_0 that element of $\{1,\ldots,T\}$ such that

$$F_{\theta_i}(t_0) - F_{\widehat{\theta}_i}(t_0) = \alpha(\theta_i,\widehat{\theta}_i). \quad (54)$$

We first have using the SLLN that $\mathbb{P}^\infty_{\theta_i}-$ almost surely,

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{\delta_i(l)\le t_0\}} = F_{\theta_i}(t_0). \quad (55)$$

Now, let $\widehat{\delta}_i^\infty$ be any sequence such that $\sum_{l=1}^\infty \mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i\}} < \infty$, i.e., $\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}$ occurs only finitely often. Combining this with (12) implies the existence of L_0 such that $\sum_{t=1}^{t_0}|f_{i,t}(L,\widehat{\delta}_i^L,\widehat{\theta}_i)| < t_0 r(L)$ for all $L \ge L_0$, which in turn implies that $|\sum_{t=1}^{t_0} f_{i,t}(L,\widehat{\delta}_i^L,\widehat{\theta}_i)| < t_0 r(L)$ for all $L \ge L_0$. Substituting (11) in this inequality and carrying out some algebra yields

$$L[F_{\widehat{\theta}_i}(t_0) - t_0 r(L)] < \sum_{l=1}^{L}\mathbb{1}_{\{\widehat{\delta}_i(l)\le t_0\}} < L[F_{\widehat{\theta}_i}(t_0) + t_0 r(L)]$$

for all $L \ge L_0$. Using (15), this implies that

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\mathbb{1}_{\{\widehat{\delta}_i(l)\le t_0\}} = F_{\widehat{\theta}_i}(t_0). \quad (56)$$

Since $\sum_{l=1}^{L}\mathbb{1}_{\{\delta_i(l)<\widehat{\delta}_i(l)\}} \ge \sum_{l=1}^{L}\mathbb{1}_{\{\delta_i(l)\le t_0\}} - \sum_{l=1}^{L}\mathbb{1}_{\{\widehat{\delta}_i(l)\le t_0\}}$ for any $L \in \mathbb{Z}_+$, dividing both sides of the inequality by L, taking the limit as $L\to\infty$, and using (56) and (55) yields

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{l}\mathbb{1}_{\{\delta_i(l)<\widehat{\delta}_i(l)\}} \ge F_{\theta_i}(t_0) - F_{\widehat{\theta}_i}(t_0) = \alpha(\theta_i,\widehat{\theta}_i)$$

$\mathbb{P}^\infty_{\theta_i}-$ almost surely, where the equality follows from (54). This establishes (28). □

Appendix C
Proof of Lemma 4

Proof. Let $\mathcal{I}_L \coloneqq \{l \le L : \mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}} = 1\}$. Denote by I_L^* the largest element of $\mathcal{I}_L$, and note that if

$$\sum_{l=1}^{\infty}\mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}} = \infty,$$

then $I_L^* = L$ for infinitely many values of L. Now,

$$\frac{1}{L}\sum_{l=1}^{L}J_p(l)\mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}} = \frac{1}{L}\sum_{l\in\mathcal{I}_L}J_p(l) \ge \frac{1}{L}J_p(I_L^*),$$

and so

$$\limsup_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}J_p(l)\mathbb{1}_{\{E_i(l,\widehat{\delta}_i^l,\widehat{\theta}_i)\}} \ge \limsup_{L\to\infty}\frac{J_p(I_L^*)}{L} = \infty,$$

where the last equality follows from (13) and the fact that $I_L^* = L$ for infinitely many values of L. □