Optimal Utilization of Storage and the Induced Price Elasticity of Demand in the Presence of Ramp Constraints

Ali Faghih † Mardavij Roozbehani ‡ Munther A. Dahleh ♯

***Abstract*—This paper is concerned with optimal utilization of storage, characterization of the economic value of storage in the presence of ramp-rate constraints and stochastically-varying electricity prices, and characterization of the price elasticity of demand induced by optimal utilization of storage. The ramp constraints limit the charging and discharging rate of storage, and can be due to the physical limitations of the storage device or the power lines. Such constraints make analytical characterization of optimal policies particularly difficult. In this paper, the optimal utilization problem is addressed in a finite-horizon stochastic dynamic programming framework, and an analytical characterization of the value function along with recursive formulas for computation of the associated optimal policy are derived. It is shown that the value function associated with the dynamic programming problem is a piecewise linear convex function of the storage state, i.e., the amount of stored energy. Furthermore, while the economic value of storage capacity is a non-decreasing function of price volatility, it is shown that due to finite ramping rates, the value of storage saturates quickly as the capacity increases, regardless of price volatility. Finally, it is shown that optimal utilization of storage by consumers could induce a considerable amount of price elasticity, particularly near the average price.**

***Index Terms*—Value of Storage, Ramp Constraints, Price Elasticity of Demand, Stochastic Dynamic Programming**

I. Introduction

Understanding the implications of optimal management of storage on the characteristics of supply and demand, and the economic and operational limitations of storage induced by ramp constraints is of practical importance to various entities, from consumers to system operators to investors in smart grid technologies. Hence, there is a need for development of econometric models and characterization of the response of a storage system to real-time price signals. This paper seeks to provide such characterization by presenting a model for optimal utilization of ramp-constrained storage in response to stochastically-varying electricity prices. The problem of optimal management of storage is formulated in a finite-horizon dynamic programming framework, and analytical expressions are given for the optimal policy and the associated value function. The effects of physical ramp constraints on optimal management of storage, and also, the economic value of storage, as well as the price elasticity of demand (PED) induced by storage are analyzed within the same mathematical framework.

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Availability of econometric models of storage management such as the one presented in this paper are particularly important for system operators who need to maintain stability and guarantee reliability of the system. For instance, it was shown in [9] that in power grids with information asymmetry between consumers, producers, and system operators, the stability and robustness of the system to disturbances are greatly affected by the consumers' real-time valuation of electricity, and their response to the real-time price. It was shown that under real-time pricing, instability and high price volatility in the resulting feedback system can be associated with high PED.

The existing literature covering various aspects of energy storage is extensive. Bannister and Kaye in [1] focus their study on optimizing the operation of a single storage connected to a general linear memoryless system in the presence of ramp constraints. However, in their model, the objective function is deterministic and known a priori. Also, Lee and Chen [7] study industrial customers with time-of-use rates and use dynamic programming to determine optimal contracts and optimal sizes of battery storage systems for such consumers. Their work, like ours, pays special attention to the economic value of storage; however, they use a deterministic approach and ignore ramp constraints.

Several other works such as [2] and [4] have studied the impacts of energy storage on the economics of integration of renewable sources. In particular, in [4], a stochastic dynamic programming framework is used to study the optimal storage investment problem through characterization of optimal sizing of energy storage for efficient integration of renewable sources. In contrast to [4], we explicitly include ramp constraints in our model, and highlight the effects of ramp constraints on the value of storage. Another related work is [2], in which Bitar et al. study the impact of energy storage capabilities on revenue of a wind power producer. Their formulation of the underlying stochastic dynamic programming problem is somewhat similar to ours; however, their focus on integration of renewable sources makes the key issues of interest and hence the context and findings of the two studies different.

The contributions of this paper are summarized as follows: First, we propose a model for optimal utilization of storage in the presence of ramp constraints. We analytically characterize the solution, and show that at each instant of time, the value function is a piecewise linear convex function of the storage state. Second, we show that although the economic value of storage increases with price volatility, due to finite ramping rate, the value of storage saturates quickly as the capacity increases. Finally, we show that optimal scheduling of storage may induce a considerable amount of price elasticity, particularly near the average price. While the demand for electricity has often been considered to be highly inelastic, the existing literature on price elasticity are mostly based on empirical evidence and qualitative reasoning, see, for instance, [5], [6], [10], and [3]. In this paper, we address price elasticity in a quantitative framework. To the best of our knowledge, this paper is the first to characterize PED induced by storage through an input-output model of response to prices based on optimal control policies in the presence of ramp constraints.

The remainder of this paper is organized as follows: In Section II, we introduce the dynamic model of utilization of storage. In Section III, we present the optimal policies for the storage management problem and analytically characterize the corresponding value function. We report our findings on the economic value of storage and PED in Sections IV and V respectively, and conclude in Section VI.

II. A Dynamic Model of Consumer Behavior

A. Notation

The set of positive real numbers (integers) is denoted by $\mathbb{R}_+$ ($\mathbb{Z}_+$), and non-negative real numbers (integers) by $\overline{\mathbb{R}}_+$ ($\overline{\mathbb{Z}}_+$). The notation for negative real and/or integer numbers is similar. The probability mass function (PMF) of a random variable Λ is denoted by P_Λ, and the cumulative distribution function (CDF) is denoted by F_Λ. We will simply use P and F when there is no ambiguity.

B. The Model

In this section, we develop a dynamic model of storage management in the presence of stochastically-varying price signals. We formulate the storage management problem as an inventory control problem over a finite horizon.

1) The Decisions: The decision set of the consumer (or storage owner) at each discrete instant of time $k \in \overline{\mathbb{Z}}_+$ is characterized by a pair

$$(v_k^{\text{in}}, v_k^{\text{out}}) \in [0, \overline{v}^{\text{in}}] \times [0, \overline{v}^{\text{out}}] \tag{1}$$

where, v_k^{in} and v_k^{out} are, respectively, the amount of electricity that the consumer injects in, or withdraws from the storage. The corresponding upper bounds ($\overline{v}^{\text{in}}$ and $\overline{v}^{\text{out}}$) represent the physical ramp constraints on storage. Also, $v_k = v_k^{\text{in}} - v_k^{\text{out}} \in [-\overline{v}^{\text{out}}, \overline{v}^{\text{in}}]$ denotes the net consumption.

2) The Price: The price process λ is assumed to be an exogenous Markovian process driven by an independently distributed random process $\mathbf{w}_k$ according to

$$\lambda_{k+1} = g_k(\lambda_k, w_k)$$

where the functions g_k and the distributions of w_k are assumed to be known for each k. It is assumed that at the beginning of each time interval $[k, k+1]$, the random variable λ_k is materialized and revealed to the consumer. A particular scenario where this model is readily applicable is where the distributions of the prices for the next 24 hours are estimated based on the day-ahead market. In this case, we may choose $g(\lambda_k, w_k) = w_k$, where the distribution of w_k is known for each k. We assume that the prices are distributed between $\lambda_{\min}$ and $\lambda_{\max}$, such that $0 \leq \lambda_{\min} < \lambda_{\max}$, with mean $\overline{\lambda}$. We also assume that the feed-in and usage tariffs are the same, i.e., λ_k is the price per unit for both consumption (corresponding to $v_k \geq 0$) and sell-back (corresponding to $v_k \leq 0$), and there are no transaction costs.

3) The States: The storage state is characterized by a variable

$$s_k \in [0, \overline{s}] \tag{2}$$

where s_k is the amount of energy stored, and $\overline{s}$ is the upper bound on storage capacity. The state s_k evolves according to:

$$s_{k+1} = \beta s_k + \eta^{\text{in}} v_k^{\text{in}} - \eta^{\text{out}} v_k^{\text{out}} \tag{3}$$

where $\beta \leq 1$ is the decay factor, $\eta^{\text{in}} \leq 1$ and $\eta^{\text{out}} \geq 1$ are charging and discharging efficiency factors[1]. The idealized model of the dynamics of storage can be written as:

$$s_{k+1} = s_k + v_k, \qquad v_k \in [-\overline{v}_{\text{out}}, \overline{v}_{\text{in}}] \tag{4}$$

which corresponds to $\beta = 1$, $\eta^{\text{in}} = 1$, and $\eta^{\text{out}} = 1$.

4) Penalty: There is a penalty $h_k(s_k)$ associated with storage, where the sequence of functions $h_k : \overline{\mathbb{R}}_+ \mapsto \overline{\mathbb{R}}_+$ are assumed to be nonnegative and monotonic.

5) The Optimization-Based Model of Ideal Storage: Since our goal in this paper is to develop tractable models that effectively highlight the important structural features of consumer behavior, we will adopt the idealized model of storage. The ideal storage management problem can be formulated as a finite-horizon dynamic programming problem as follows:

$$\begin{aligned} \min \quad & \mathbf{E}\left[\sum_{k=0}^{N} h_k(s_k) + \lambda_k v_k\right] \\ \text{s.t.} \quad & s_{k+1} = s_k + v_k \\ & \lambda_{k+1} = g_k(\lambda_k, w_k) \\ & s_k \in [0, \overline{s}] \\ & v_k \in [-\overline{v}^{\text{out}}, \overline{v}^{\text{in}}] \end{aligned} \tag{5}$$

Remark 1. *We formulate and solve the storage problem for the finite-horizon case, and assign a value of $\hat{\lambda}$, the mode of the price distribution, to each unit of energy left in storage by the end of the time horizon.*

III. Main Results: The Optimal Policy

In this section, we characterize the optimal policy for problem (5) based on principles of stochastic dynamic programming. We show that under some technical assumptions, at each instant of time, the value function is a convex piecewise linear function of the storage state.

[1]The efficiency factors and the ramp rates might in general be complicated functions of the operating point, i.e., the storage level.

Definition. *Given a probability mass function P, let Θ and ψ be a pair of maps from the set of all subsets of $\overline{\mathbb{R}}_+$ to $\overline{\mathbb{R}}_+$ defined according to*

$$\Theta : I \mapsto \sum_{\theta \in I} \theta P(\theta), \qquad \forall I \subset \overline{\mathbb{R}}_+$$
$$\psi : I \mapsto \sum_{\theta \in I} P(\theta), \qquad \forall I \subset \overline{\mathbb{R}}_+.$$

Given $\overline{v} \in \mathbb{R}_+$, and maps Θ and ψ as defined above, let $\Phi_{\overline{v}}$ be a map from the set of all subsets of $\overline{\mathbb{R}}_+$ to $\mathbb{R}$ defined according to

$$\Phi_{\overline{v}} : I \mapsto \overline{v}\,(\Theta - \rho\psi)\, I, \qquad \forall I \subset \overline{\mathbb{R}}_+,$$

where

$$\rho = \inf I.$$

For instance, Φ_1 maps an interval (a,b) to $(\Theta - a\psi)(a,b)$.

Theorem 1. *Consider the finite-horizon storage control problem (5) with $\overline{v}^{in} = \overline{v}^{out} = \overline{v}$, and $\overline{s} = n\overline{v}$ for some $n \in \mathbb{Z}_+$, and i.i.d price process λ, i.e., $g_k(\lambda_k, w_k) = w_k$ for all k. Furthermore, assume that the penalty functions $h_k : [0,\infty) \to [0,\infty)$, $k = 0,..,N$ are piecewise linear non-decreasing convex functions of the form:*

$$h_k(s) = h_k^i s + c_k^i,\ \ s \in [i\overline{v}, (i+1)\overline{v}),\ \ i \in \overline{\mathbb{Z}}_+ \tag{6}$$

Then

(i) The optimal policy is characterized by:

(a) If $0 \le s_k < \overline{v}$, then

$$v_k^* = \begin{cases} -s_k & \text{if} \quad t_{k+1}^0 < \lambda_k \\ \overline{v} - s_k & \text{if} \quad t_{k+1}^1 < \lambda_k \le t_{k+1}^0 \\ \overline{v} & \text{if} \quad \lambda_k \le t_{k+1}^1 \end{cases}$$

(b) If $s_k \ge \overline{v}$, so that $s_k \in [i\overline{v}, (i+1)\overline{v})$, for some $i \in \{1, 2, ..., n-1\}$, then

$$v_k^* = \begin{cases} -\overline{v} & \text{if} \quad t_{k+1}^{i-1} < \lambda_k \\ i\overline{v} - s_k & \text{if} \quad t_{k+1}^{i} < \lambda_k \le t_{k+1}^{i-1} \\ (i+1)\overline{v} - s_k & \text{if} \quad t_{k+1}^{i+1} < \lambda_k \le t_{k+1}^{i} \\ \overline{v} & \text{if} \quad \lambda_k \le t_{k+1}^{i+1} \end{cases}$$

and the thresholds are given by the recursive equations:

$$\begin{aligned} & t_N^i = \hat{\lambda}, \qquad i \in 0,1,2,...,n-1 \\ & t_N^i = -h_N^i, \qquad i \ge n \end{aligned}$$

for $k < N$:

$$\begin{aligned} t_k^0 &= t_{k+1}^1 + \Phi_1(t_{k+1}^1, \lambda_{\max}] - h_k^0 \\ t_k^i &= t_{k+1}^{i-1} - h_k^i + \Phi_1(t_{k+1}^{i+1}, t_{k+1}^{i-1}] \\ &\quad + (t_{k+1}^{i+1} - t_{k+1}^{i-1}) F\left(t_{k+1}^{i-1}\right), \qquad i \ge 1 \end{aligned}$$

(ii) The value function is a piecewise linear convex function of the form:

$$V_k(s) = -t_k^i s + e_k^i,\ \ s \in [i\overline{v}, (i+1)\overline{v}), i \in \overline{\mathbb{Z}}_+ \tag{7}$$

where $t_k^{i+1} \le t_k^i$ for all k and i.

Proof: The proof is omitted due to space limitation. Please see the full version on arXiv. ■

Remark 2. *We will use e_0^0 as a function of the distribution to characterize the value of storage. The e_k^i parameters of the value function are given by the following recursive equations:*

$$\begin{aligned} & e_N^i = 0, \qquad i \in \{0,1,2,...,n-1\} \\ & e_N^i = \overline{s}(t_N^i - \hat{\lambda}), \qquad i \ge n \end{aligned}$$

for $k < N$:

$$\begin{aligned} e_k^0 &= c_k^0 + e_{k+1}^0 + (\overline{v}\lambda_{\min} + e_{k+1}^1 - e_{k+1}^0 - \overline{v}t_{k+1}^1) F(t_{k+1}^0) \\ &\quad + \Phi_{\overline{v}}[\lambda_{\min}, t_{k+1}^0] \\ e_k^i &= c_k^i + \overline{v}\overline{\lambda} + f(t_{k+1}^{i-1}, t_{k+1}^i, t_{k+1}^{i+1}, e_{k+1}^{i-1}, e_{k+1}^i, e_{k+1}^{i+1}) \\ &\quad + g(t_{k+1}^{i-1}, t_{k+1}^i, t_{k+1}^{i+1}),\ \ i \ge 1 \end{aligned} \tag{8}$$

where the functions f and g are given by

$$\begin{aligned} f(\cdot) &= e_{k+1}^{i-1} - \overline{v}t_{k+1}^{i+1} + (e_{k+1}^i - e_{k+1}^{i-1}) F(t_{k+1}^{i-1}) \\ &\quad + (e_{k+1}^{i+1} - e_{k+1}^i) F(t_{k+1}^i) \\ g(\cdot) &= (i+1)\Phi_{\overline{v}}(t_{k+1}^{i+1}, t_{k+1}^i] + i\Phi_{\overline{v}}(t_{k+1}^i, t_{k+1}^{i-1}] \\ &\quad - \Phi_{\overline{v}}(t_{k+1}^{i-1}, \lambda_{\max}] - \Phi_{\overline{v}}(t_{k+1}^{i+1}, \lambda_{\max}]. \end{aligned}$$

Remark 3. *The results in Theorem 1 are expressed for discrete probability distributions, but they extend naturally to continuous distributions under some technical assumptions.*

Remark 4. *The upper bound $\overline{s}$ on the storage capacity is enforced by choosing h_k^i in (6) sufficiently large (i.e. $h_k^i > \lambda_{\max}$) for $i \ge n$, so that it would never be optimal to store energy beyond $\overline{s}$.*

IV. The Value of Storage

We define the expected economic value of storage, or simply the value of storage, as the negative of the cost of the optimal value of problem (5), and we denote it by $\mathcal{V}$. Therefore, $\mathcal{V} = -V_0(s_0)$. Throughout this section we assume that N is fixed and $s_0 = 0$, which means that the consumer starts with an empty storage. This implies, using (7), that the value of storage becomes:

$$\mathcal{V} = -V_0(0) = -e_0^0. \tag{9}$$

which can be computed using the recursive equations in (8). For convenience, let us define the following distributions.

Definition. *A low-high distribution is a mixture of two impulses, where the low price (L) has probability $9/10$ and the high price (H) has probability $1/10$. In our computations, this probability distribution will serve as a proxy for the case of an electricity market with somewhat frequent price spikes. We also define a discrete uniform distribution with support between non-negative integers a and b. Letting $M = b - a + 1$, we have $P(\theta) = 1/M$ for $a \le \theta \le b$, $\theta \in \overline{\mathbb{Z}}_+$, and $P(\theta) = 0$ otherwise. Note that throughout this paper,* *for the discrete uniform distribution, we use the mean of the distribution as its mode, i.e. we assume that $t^i_N = \hat{\lambda} = \overline{\lambda}$.*

In this section, using the above definition, we consider the following classes of distributions:

a) Discrete uniform distribution, with fixed mean $\overline{\lambda} = 80$,
b) Low-high distribution, with fixed mean $\overline{\lambda} = 80$.

For each of these distributions, we fix all quantities in our model other than σ, the standard deviation of price distribution, and n, the ratio of storage capacity $\overline{s}$ to physical ramp constraint of storage $\overline{v}$. We vary $n = \overline{s}/\overline{v}$ by fixing $\overline{v}$ and changing $\overline{s}$. Note that as defined in Theorem 1, n only takes on integer values. Using the fixed quantities $N = 30$, $\overline{v} = 10$, and $\overline{\lambda} = 80$, we examine how the normalized $\mathcal{V}$ (normalized by the time horizon N) varies as a function of σ and n, once for the case of no storage penalties, and another time in the presence of storage penalties.

A. Without Storage Penalties

Herein, we set $h^i_k = 0$, for all $i \leq n-1$ and $k \leq N$, so that there is no penalty on storing energy up to capacity. Then, for a fixed time horizon, we examine how $\mathcal{V}/N$ varies with σ and n, for each of the following price distributions:

1) Discrete uniform distribution: Figure 1 illustrates how $\mathcal{V}/N$ changes with σ and n, for the discrete uniform distribution. The plots show that the value of storage increases linearly with σ. As one would expect, the value of storage also increases as the storage capacity increases. However, it is interesting to note that for a fixed standard deviation, the value of storage saturates fairly quickly as a function of n. Hence, for a given time horizon, a fixed ramp constraint, and a fixed σ, there exists a certain storage capacity beyond which the value of storage will no longer change noticeably. Also note that the magnitude of n at which this saturation occurs is an increasing function of σ, implying that the optimal storage capacity increases with price volatility.

2) Low-high distribution: As can be seen in Figure 2, saturation of the value of storage occurs much more quickly than the case of the discrete uniform distribution. This implies that the price distribution has a considerable impact on the value of storage. However, the value of storage is still a linear function of the standard deviation, just like the case of the discrete uniform distribution.

B. With Storage Penalties

This time we set the storage penalty to $h^i_k = 0.1\hat{\lambda}$, for all $i \leq n-1$ and $k \leq N$.

1) Discrete uniform distribution: As shown in Figure 3, in contrast to the previous case, the plots show that the value of storage is no longer a linear function of σ, though, it is still an increasing function of σ. The observation here is that the value of storage saturates more quickly than the case with no penalties, which makes intuitive sense considering the high cost of keeping a lot of energy in storage. Also, note that for this case, the marginal value of storage with respect to σ is almost zero for low values of σ.

2) Low-high distribution: As shown in Figure 4, the value of storage saturates even more quickly than the case of no penalties. Also, the marginal value of storage with respect to σ is almost zero even for relatively large values of σ.

An interesting observation in this section is that in the presence of ramp constraints, several distributed storage systems would be more profitable than one large storage system of equal ramp constraint and total capacity, due to the quick saturation of $\mathcal{V}$ as n increases.

V. Price Elasticity of Demand

In the previous sections, we developed a model that characterizes an individual consumer's optimal policy for managing storage and the associated economic value of storage. In this section, we will introduce a simple model of aggregation, where each individual uses the storage management model (5). We give consumers randomized initial states, and simulate their response. In particular we compute their consumption, and cluster them as a function of the real-time price in order to characterize the PED.

A. Aggregation Model

We denote the number of consumers by L, and specify the aggregation model as follows. We assume that there is a fixed time horizon for all consumers, which we denote by $\overline{N}$. At each time $k \in \{0, \cdots, \overline{N}\}$, all consumers are given the same price signal λ_k. However, to model random initial states, consumer $j \in \{1, \cdots, L\}$ starts at time $k = 0$ with a random initial state that is uniformly distributed over $[0, \overline{s}]$. We denote the local state for each consumer at time k by s^j_k. The total consumption of all consumers at time k is the ensemble average of the individual v^j_k values.

The PED is defined as the ratio of the percentage change in demand to the percentage change in price. To make the notion of PED accurate, one needs a measure of consumption that depends only on price. In our dynamic model, however, the consumption depends on price, stage, state, storage capacity, and ramp constraint. Quick saturation of the value of storage with storage capacity for a fixed ramp rate (as shown in Section IV) makes it reasonable to use the same storage capacity for all consumers within the same sector (e.g. the residential sector). Moreover, we can eliminate state-dependence by taking expectations. In particular, we can define:

$$v^j(k, \lambda) = \mathbf{E}_{\lambda_0, \lambda_1, \ldots, \lambda_{k-1}, s^j_0} \left[v^{j*}_k | \lambda_k = \lambda \right],$$

In order to eliminate stage-dependence, we think of the consumption-measuring observer as sampling a random time τ uniformly over $\{0, \cdots, \overline{N}\}$. By averaging over this randomness, we maintain dependence on price alone:

$$v_{\text{aggr}}(\lambda) = \frac{1}{L} \sum_{j=1}^{L} \mathbf{E}_\tau \left[v^j(\tau, \lambda) \right],$$

which is easily captured in numerical simulations by clustering real-time prices, and averaging over each cluster.

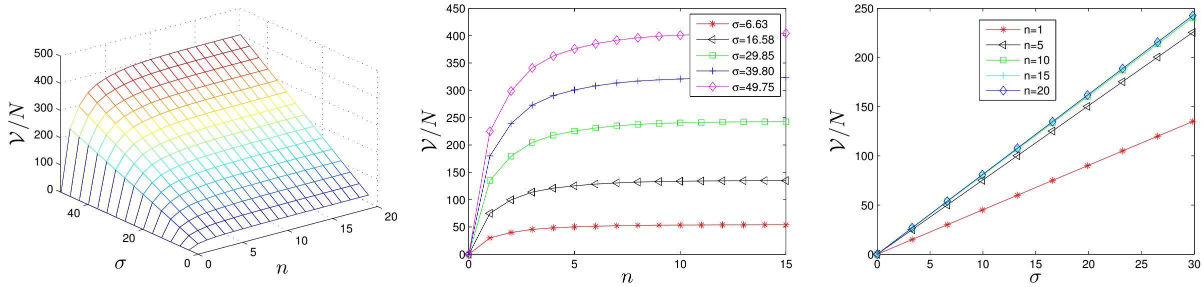

Fig. 1. $\mathcal{V}/N$ vs. σ and n, in 3-D (left), and its 2-D projections for a few samples (middle and right), without penalties, using a discrete uniform distribution.

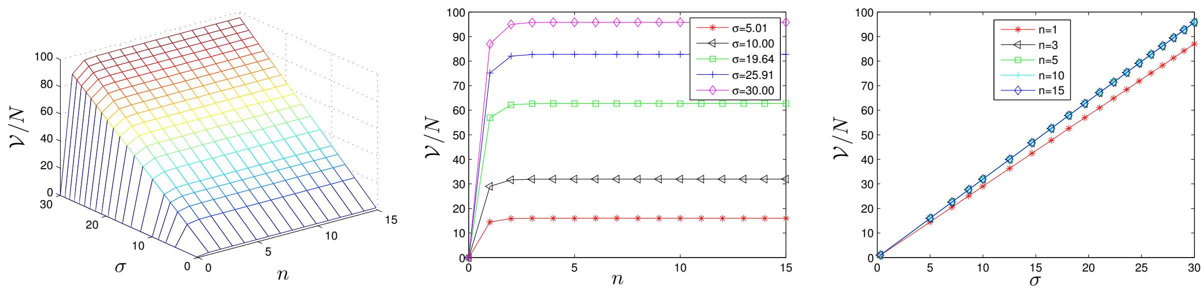

Fig. 2. $\mathcal{V}/N$ vs. σ and n, in 3-D (left), and its 2-D projections for a few samples (middle and right), without penalties, using a low-high distribution.

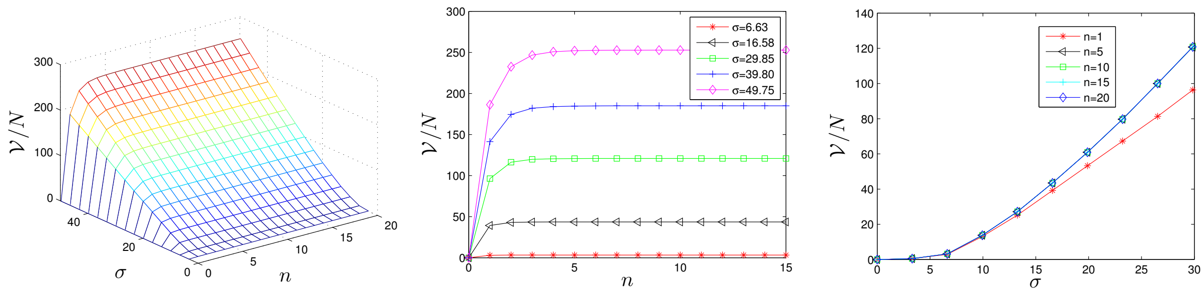

Fig. 3. $\mathcal{V}/N$ vs. σ and n, in 3-D (left), and its 2-D projections for a few samples (middle and right), with penalties, using a discrete uniform distribution.

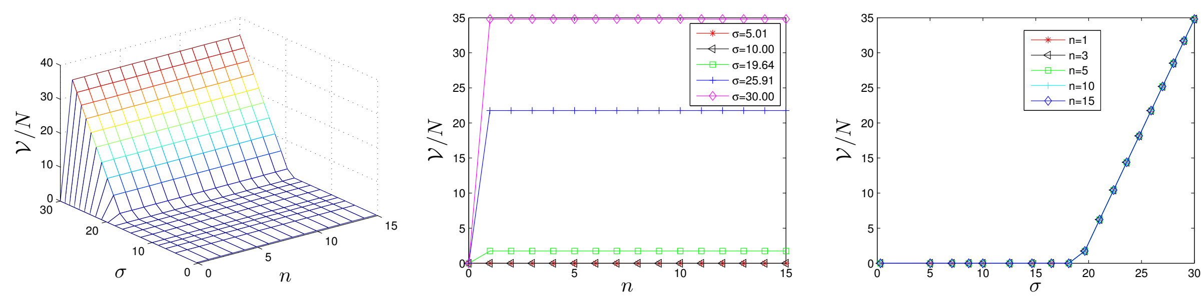

Fig. 4. $\mathcal{V}/N$ vs. σ and n, in 3-D (left), and its 2-D projections for a few samples (middle and right), with penalties, using a low-high distribution.

B. Simulations

1) Aggregation Parameters: In these numerical simulations we assume that the number of consumers is $L = 30$, and we average over 20 random instances of price and consumer initial states. We set $\overline{N} = 288$, which corresponds to a period of 24 hours, where real-time prices are updated once in every 5 minutes. They each implement the optimal policy given in Theorem 1. We simulate a discrete uniform distribution with mean $\overline{\lambda} = 10$. Based on our results in Section IV, for these model parameters, a storage capacity of $\overline{s} = 5\overline{v}$ is a reasonable choice for all consumers.

We examine two different scenarios:

(a) $h_k^i = 0$ for all $k \leq N$ and $i < n$, and

(b) $h_k^i = 0.1\overline{\lambda}$ for all $k \leq N$ and $i < n$.

Figure 5 illustrates how the aggregate demand changes as a function of price for (a) and (b), using the discrete uniform price distribution.

C. Interpretation

As the plots for both cases suggest, the aggregate demand seems to be more responsive to prices that fall in the mid-portion of the price range. For the case of no storage penalties, this portion serves as a relatively steep transition region, in which the consumer quickly switches from the "buy it all" policy to the "sell it all" policy. The situation is slightly different when storage penalties are imposed.

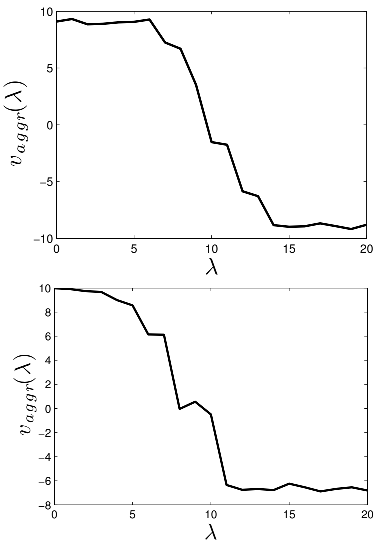

Fig. 5. (a) Aggregate demand vs. price, using the discrete uniform price distribution, without storage penalties. (b) Aggregate demand vs. price, using the discrete uniform price distribution, with storage penalties.

Considering the cost of storing energy, the "buy it all" region has become narrower, and the transition region has instead become wider. The selling policy has practically been reduced to "sell it all if prices are above average" with the exception that due to the cost of keeping a lot of energy in inventory, the expected sell-back to the grid has become close to half of the ramp constraint, for practically any price that is greater than average.

To characterize PED in a more quantitative way, one needs to bear in mind that the overall PED should have the firm component of demand in it. Recall the definition of PED:

$$PED = \frac{\Delta d/d}{\Delta\lambda/\lambda}$$

where d denotes demand. Here, we have $d = d^f + v_{\text{aggr}}(\lambda)$, where d^f denotes the firm demand. So, the overall PED depends on how much storage we have compared to the firm demand. More storage yields higher elasticity. Similarly, a smaller firm demand results in higher PED. Also, note that the PED is almost zero for prices that are considerably larger or smaller than the mean price, and only in the mid-portion of the plots (i.e. around the mean price) we observe a substantial PED. For instance, in the above simulations for scenario *(a)*, if we set the firm demand equal to 3 times the ramp constraint (i.e. $d^f = 3\overline{v}$), then the PED around the mean price (i.e. around $\lambda = 10$) would be about -0.9, whereas setting $d^f = 10\overline{v}$ yields a PED of about -0.26 around the mean price. Also, in the above simulations for scenario *(b)*, if we set $d^f = 3\overline{v}$, then the PED around the mean price would be -1.2, whereas setting $d^f = 10\overline{v}$ yields a PED of about -0.34 around the mean price.

Both of these scenarios confirm that a lower fixed demand relative to storage level yields higher elasticity. We also observe that the PED at the mean price is higher in the presence of storage penalties compared to case of no penalties, because when we have penalties, the optimal policy becomes the "sell it all" policy right after the mean price, which causes a jump in the aggregate demand.

Although we present a consumer-aggregate model, because the stochastic behavior of each user is the same, the ensemble average provided in this section is equivalent to a single-user expectation. Moreover, since our time horizon is fairly long, the initial state s_0 only affects the optimal policy of a consumer for the first few stages. Hence, within a short period after the initial time, the states for all consumers will become the same.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed a dynamic model of storage management. We derived the optimal policy for the storage problem with ramp constraint, and showed that the value function is a convex piecewise linear function. We provided analytical expressions for the expected economic value of storage, and illustrated how the economic value of storage varies as a function of price volatility and the ratio of storage capacity to physical ramp constraint.

An immediate observation that one could make in our results is that if all the consumers optimally schedule their utilization of storage capacity in the presence of bi-directional meters, a considerable amount of power will be fed back into the grid when the prices are above the mean price, and the direction is reversed for prices that are below the average price. This implies that the consumers' utilization of storage capacity may need to be regulated by the system operator to maintain system balance and stability.

Our future work includes embedding this storage model into a feedback loop under various scenarios of bi-directional metering to study long term effects of storage on stability of electricity markets and integration of renewable sources.

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