

Identification in the presence of bounded low-correlated noise.

S. R. Venkatesh I. Chen M.A. Dahleh J.N. Tsitsiklis
Laboratory for Information and Decision Systems

Massachusetts Institute of Technology
Cambridge, MA 02139

In this paper, we consider the problem of identification of discrete-time, single-input-single-output linear time-invariant systems. The prior information about the unknown plant is that it belongs to a certain set of LTI systems. The plant will be identified using input-output experiments, where the input can be chosen freely and the observed output is corrupted by an additive disturbance which is assumed to be bounded and has low-correlation. This set of disturbance includes uniformly bounded sequences that satisfy a time-averages correlation condition. The motivation for using such a disturbance set is that the disturbances resemble stochastic white noise processes. In particular, a white noise signal belongs to this set with high probability if the bound on the correlations approaches zero at a certain rate. Also, given any stable linear time-invariant system, the variance of the output for a white noise input is given by the H_2 norm of the system. It turns out that the ℓ_2 induced norm of any stable LTI system over the above set of inputs approximates asymptotically (in a precise sense) the H_2 norm of the system.

This paper analyzes the problem of worst-case identification in the presence of the above set of disturbances. Several results are shown. Firstly, we show that the worst-case diameter of uncertainty (which provides an upper bound on the worst-case error) decreases to zero as the correlation bound approaches zero. Here the correlation bound is assumed to be arbitrary and not dependent on the length of the experiment. Next we compute sample complexity of identification of FIR systems in this setting and show that for every finite non-zero correlation bound, the length of the input required to get within a factor of the optimal error is exponential in the number of parameters to be identified. A lower bound on the

sample complexity is derived that depends on both the allowable error and the correlation bound.

Finally, we exploit the fact that the correlation bound can be allowed to be a function of length of measurements without sacrificing the fact that the disturbances contains white noise. We show that the radius of uncertainty can be reduced to zero for certain model sets under this setting. Furthermore, the size of the input required to reduce the uncertainty set to an a priori assigned number is polynomial in the size of the parameters. Moreover, the procedure works for all cases considered in the classical stochastic set up. Thus these results parallel the standard stochastic results without any probabilistic assumptions. Instead of getting confidence intervals, one gets hard bounds on the error, and the complexity is not much higher. The pertinent literature can be found in the following references.

References

- [1] M. A. Dahleh, T. V. Theodosopoulos and J. N. Tsitsiklis, *The sample complexity of worst case identification of linear systems, System and Control letters, Vol. 20, No. 3, March 1993*
- [2] Fernando Paganini, *White noise rejection in a deterministic setting, CDC93*
- [3] T. Soderstrom, P. Stoica *System Identification, Prentice Hall 1989*
- [4] D. N. C. Tse, M. A. Dahleh, J. N. Tsitsiklis *Optimal identification under bounded disturbances, IEEE Trans on Automatic Control, Vol. 38, No. 8, August 1993*
- [5] K.H. Chen, *Worst case identification for bounded and low correlated disturbance, S.M. thesis, M.I.T.*
- [6] S. R. Venkatesh, M.A. Dahleh *Classical Identification in a deterministic setting, LIDS Report P-2296*