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A New Framework for Iterative Identification and Control

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Abstract

In this paper we present a new framework for iterative modeling and control. We begin by describing the unknown process with an uncertain model whose parameterization depends on prior information, available control design tools and other modeling preferences. The next step is an iterative procedure for refining the uncertainty set via robust control based model invalidation and can be viewed as a systematic way of efficiently searching for a controller delivering a certain desired level of performance to the unknown process. As a result, either the performance goal will be met or the entire uncertainty set will be invalidated in accordance with our modeling and control method prejudice. An iterative scheme based on a fixed pole model structure and rank one mixed μ synthesis will be described in detail and a specific example will be used to illustrate the ideas.

1. Introduction

Over the past decade, there has been much research activity in the area of worst–case, or control–oriented system identification. The motivation can be attributed to new advances in robust control theory which did not interface well with existing theory of classical system identification. The main focus of this research has been the design of algorithms that yield nominal models along with measures of uncertainty which are well suited for robust control design [8, 7, 21, 17, 11]. Unfortunately, these worst–case algorithms tend to provide error bounds which are very conservative in practice [10] and are therefore of limited utility. This is one motivation for the area of iterative identification and control which has recently gained attention in the control community. Several researchers have been working on the connections between identification and control [1, 23, 24, 6, 14, 19, 20, 13, 12]. This work has typically been in the spirit of adaptive control. In other words, a sequence of nominal models is being identified, while a sequence of corresponding controllers (usually robust) is being designed for these models. The hope is that the models are, in some sense, getting closer to the unknown process and the performance is improving.

The more recent formulation of Dahleh and Doyle [2] uses a somewhat different philosophy. Here, the goal is to observe some experimentally generated finite set of data and find a controller that meets a given performance specification for the unknown process. The model is thought of as a tool which is chosen based on the designer's preferences of control design techniques and ways of explaining the observed data. In this sense, the chosen model parameterization is good only if a controller designed for this model can also achieve good performance with the unknown process. On the other hand, if a controller delivers good performance with the model, yet fails to meet the performance specifications with the unknown process, this model is considered to be a poor description of the process and should be invalidated. In this way, a conservative model set can be effectively shrunk until the remaining elements can deliver a controller which will achieve the desired performance specifications on the actual process, or the whole set is invalidated.

In this paper we give a concrete example of an iterative scheme that was presented in [16]. This iterative scheme is based on a fixed pole model (FPM) and the rank one mixed μ synthesis (ROS) robust control technique. The following sections discusses the model and the robust control technique. This is followed by a detailed description of the iterative scheme based on the FPM and ROS, and some of the computations associated with it. Finally, a specific example of the ROS based iterative scheme is considered in Section 5.

2. Selection of Model Parameterization

In general, the selection of the model parameterization is a process that requires engineering insight as well as careful consideration of available robust control techniques and available information about the process to be controlled. Currently there are few robust control methods available and the existing methods incorporate only special types of performance objectives and uncertainty structures [4, 3, 22, 18, 5, 9]. All of these

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design methods can accommodate unmodeled dynamics as uncertainty and norms of weighted transfer functions as design specifications. The mixed–μ (rank one) synthesis method of Rantzer and Megretskii [18] can also nonconservatively accommodate parametric uncertainty models (having some structural restrictions). Once the desired model structure, or parameterization is chosen, the problem is to efficiently map the prior information about the unknown process into a model set having the desired structure. This step can be extremely difficult and so the structure of the prior information can significantly influence the choice of model parameterization.

This model set transformation step can be made more rigorous by defining the original prior information set to be the set of models consistent with the priors

$$\mathcal{M}_{\text{prior}} = \{P(a) : a \in A_p \subset \mathbb{R}^n\} \tag{1}$$

where the prior information is embedded in A_p and the functional dependence of $P(a)$ on a. Note that any unmodeled dynamics are also contained in $\mathcal{M}_{\text{prior}}$. Given the desired model structure parameterization, $G(\theta, \Delta)$, the goal is to find the smallest $\epsilon \geq 0$ and $\Theta_o \subset \mathbb{R}^m$ such that

$$\mathcal{M}_{\text{prior}} \subseteq \mathcal{M}_{\text{des}} \equiv \{G(\theta, \Delta) : \theta \in \Theta_o, ||\Delta||_\infty \leq \epsilon\}$$

This is generally a very difficult problem to solve and some approximate solutions are examined in [15].

2.1. Fixed Pole Model for Rank One Synthesis

The ROS design method was developed by Rantzer and Megretskii in [18]. The method is limited to special model structures (i.e., SISO, MISO and SIMO with real and complex coprime factor perturbations), however, the solution is a convex optimization problem. If we also require robust performance such as minimizing a weighted sensitivity (i.e., $||W_pS||$), the uncertainty can only be in the *numerator* of the model if the rank one structure is to be maintained. The corresponding model will be referred to as the fixed pole model (FPM) and is given by

$$G(\theta, \Delta) = \frac{\sum_{k=0}^{m-1} \theta_k z^k + W\Delta}{A(z)} \tag{2}$$

where $||\Delta||_\infty \leq 1$, $\theta \in \Theta \subset \mathbb{R}^m$, W is a stable and invertible weighting function, and $A(z)$ is a stable polynomial.

When the prior information is given in terms of uncertain pole locations or other structures which are not compatible with fixed poles, the mapping of these priors into the appropriate parameter set Θ and a weighting function W can be difficult. In particular, when $W = \epsilon W_o$, one would like to compute the smallest $\epsilon > 0$ and the corresponding set Θ such that the FPM set contains all the plants given by the prior information. It can be shown that computing ϵ is equivalent to computing the n–width of the prior model set [15], and finding the corresponding parameter uncertainty set Θ is also a difficult task. For this model and the iterative scheme described in Section 4, it is important to choose W with very little conservatism (i.e., try to lump as much of the pole mismatch into the parametric uncertainty as possible) because the iterative scheme will reduce uncertainty in the parametric part, while the $W\Delta$ part will remain fixed. The problem of choosing the smallest ϵ is examined in detail in [15].

3. Rank One Mixed μ Synthesis

We first show that the fixed–pole model (FPM) having a hypercube for the parameter uncertainty set is a special case of the perturbed coprime model (PCM). The PCM is the model used in the rank one synthesis theory and is of the form (SISO case)

$$G_{\delta,\Delta} = \frac{N + \delta^T N_\delta + \Delta N_\Delta}{M + \delta^T M_\delta + \Delta M_\Delta}$$

with $N, M \in RH_\infty$, (N, M) coprime, $\delta \in \mathbb{R}^m$, $||\delta||_\infty < 1$, $\Delta \in RH_\infty$, $||\Delta||_\infty < 1$, $N_\delta, M_\delta \in RH_\infty^m$ and $N_\Delta, M_\Delta \in RH_\infty$. The fixed–pole model we want is given by

$$G_{\theta,\Delta} = \frac{B(\theta)}{A} + W\Delta$$

where $B(\theta) = \sum_k \theta_k z^k$, $\theta \in \Theta \subset \mathbb{R}^m$, Θ is a hypercube which is centered at θ_c and has side lengths $\{\eta_k\}$, and $||\Delta|| \leq 1$. This corresponds to the PCM model above with $M = 1$, $N = B(\theta_c)/A$, $N_\Delta = W$, $M_\Delta = 0$, $M_\delta = 0$, and

$$N_\delta = \frac{[\eta_0 \; \eta_1 z \cdots \eta_{m-1} z^{m-1}]^T}{A(z)}$$

We now incorporate the robust performance objective which is given by $||W_pS||_\infty \leq \gamma$. This can be transformed into a robust stability problem as follows. Define $G_a \equiv G_{\delta,\Delta}(1 - \Delta_p W_p)^{-1}$ which can also be expressed in terms of the uncertain coprime model as

$$G_a = \frac{N + \delta^T N_\delta + \Delta N_\Delta + \Delta_p N_{\Delta_p}}{M + \delta^T M_\delta + \Delta M_\Delta + \Delta_p M_{\Delta_p}} \tag{3}$$

with $N_{\Delta_p} = 0$, $M_{\Delta_p} = W_p$, and the rest of the quantities defined as above. The goal is to find the largest γ^{-1} such that G_a can be stabilized for all $||\Delta|| < 1$ and $||\Delta_p|| < \gamma^{-1}$.

The general rank one synthesis result was solved by Rantzer and Megretski [18] who derived a convex parameterization of all robustly stabilizing controllers for rank one uncertainty lying in a convex set. We will state this result in the form specialized for the FPM.

Let the nominal model, $B(\theta_c)/A$, be denoted by G and define

$$\phi(\alpha,\beta) = \sup_{\omega\in[0,2\pi]} \frac{|W_p([\alpha+\beta]G+\alpha)|}{Re\alpha - \|Re\{N_\delta[\alpha+\beta]\}\|_d - |N_\Delta(\alpha+\beta)|}$$

where α is a positive real transfer function, β is any stable transfer function, and all quantities are evaluated at $e^{i\omega}$.

4. Iterative Procedure

Having mapped the priors into a FPM set and having a solution to the scheme robust control problem, one can try using an iterative based on the FPM and ROS with a performance objective which is implied by keeping the weighted sensitivity transfer function small.

Partitioning of the model set will be performed with respect to Θ while $W\Delta$ is assumed to represent the inherent nonparametric uncertainty and will remain fixed in size. Thus, we will refer to Θ as the model set and suppress the $W\Delta$ part which is fixed for each parameter value in Θ. The iterative procedure based on ROS consists of the following steps.

1. Label the initial model set Θ_0 and set $k=0$.
2. Can the desired performance be achieved for Θ_k by some K_k? If yes, go to (4).
3. Refine Θ_k in the following way (to achieve better performance):
   (a) Find j such that the performance is most sensitive with respect to the j^{th} parameter, θ_j.
   (b) Split Θ_k along the j^{th} dimension, resulting in the two sets X_0 and X_1, with $\Theta_k = X_0 \cup X_1$.
   (c) (Skip if $k=0$) If X_0 is smaller than the smallest allowable partition size we invalidate Θ_k by decrementing k by 1, and go to (2).
   (d) Find $q \in \{0,1\}$ such that the best performance which can be achieved for X_q is better than the one for X_{1-q}. Let K_{k+1} be the controller which delivers this performance to X_q.
   (e) Set $\Theta_k = X_{1-q}$, $\Theta_{k+1} = X_q$, increment k by 1, and go to (2).
4. Connect K_k to the plant and test for performance
5. If the performance is satisfied, stop.
6. If $k>0$ invalidate Θ_k by decrementing k by 1 and go to (2). Otherwise, choose a new model parameterization and go to (1).

This procedure has several nice properties. Choosing the smallest allowable partition size to be nonzero, we are guaranteed termination in finite time. Every time a set is split, the memory requirement is only increased by one unit (containing the center and side lengths information, for example) so there is no geometric or exponential explosion in required memory. The search is optimistic, always seeking the best set in the partition.

The computationally difficult steps are steps 2, 3d, and possibly 3a. Note that in steps 2 and 3d, we are trying to synthesize controllers meeting either the desired or the best possible performance levels, with step 3d having to solve two such problems. Step 3a which computes the sensitivity of performance with respect to each parameter is fairly easy to compute in the special case of ROS and FPM. The solution is given by the following result, which we state without proof.

Lemma 4.1 *Assume that the ROS solution gives a feasible pair $(\alpha(z),\beta(z))$ as well as the worst frequency, ω_0 which maximizes the functional. Then the parameter which has the greatest impact on performance is given by $\theta_{k_{max}}$, where*

$$k_{max} = \arg\max_{k\in[0,n-1]} \left| Re\left\{ e^{i\omega_0 k}\left(\frac{\beta(e^{i\omega_0})+\alpha(e^{i\omega_0})}{A(e^{i\omega_0})}\right)\right\}\right|$$

5. Illustrative Example

In this section we present an example which illustrates the iterative scheme described in the previous sections. It is assumed that the plant is known to consist of a second order lightly damped mode with two flexible modes at higher frequencies. The lightly damped mode is known to be of the following form.

$$P(s) = \frac{1}{s^2+2\xi\omega_n s+\omega_n^2}$$

where $\underline{\omega} \le \omega_n \le \overline{\omega}$, $\underline{\xi} \le \xi \le \overline{\xi}$, and it is known that $\underline{\omega}=0.7$, $\overline{\omega}=0.8$, $\underline{\xi}=0.2$, and $\overline{\xi}=0.3$. It is known that the two other modes occur at frequencies of approximately 8 and 12 rad/s. We can discard these modes and represent them with unmodeled dynamics of the form $W\Delta$. After converting everything to discrete time ($T_{sam}=0.15$s) we model the simplified plant by the fixed pole approximation. The simplified plant is given by the following.

$$P(a;z) = \frac{0.0121z+0.0119}{z^2+a_1 z+a_0} \quad \begin{array}{lcl} a_1 & \in & [-1.9456,-1.9122] \\ a_0 & \in & [0.9274,0.9570] \end{array}$$

This means that the fixed pole model is of the form

$$G(\theta,\Delta) = \frac{\sum_{k=0}^{m-1}\theta_k z^k + (\epsilon^* + DW)\Delta}{(z^2-1.9289z+0.9422)^{\lfloor m/2\rfloor}}$$

where ϵ^* can be obtained from the results in [15] and $D(z)$ is the denominator polynomial $(z^2-1.9289z+$

$0.9422)^{\lfloor m/2 \rfloor}$. The ϵ^* represents the error between the FPM and simplified plant, while the DW term is the error between the full order and simplified plant.

We assume that the actual plant is given by

$$P(z) = \frac{0.00181z^5 - 0.0026z^4 + 0.0055z^3 + 0.0087z^2 + 0.0005z + 0.0053}{z^6 - 2.4015z^5 + 2.5568z^4 - 1.8369z^3 + 0.9620z^2 - 0.4987z + 0.2362}.$$

For visual presentation purposes we consider a fixed pole model of order $m = 2$. The corresponding ϵ^* is 0.0084. The ideal performance objective is assumed to be good tracking of certain duration step inputs, however, for the robust control design we will use small l_2 gain of the weighted sensitivity transfer function as the design objective. We consider a weighting function which will allow designs of fairly demanding bandwidth (i.e., beyond the first lightly damped mode). This weighting function is given by $W_p = \frac{z-.2152}{z-.9984}$.

5.1. Iterative Scheme Simulation Results

We now demonstrate a few examples of the iterative scheme. We take the two dimensional parametric uncertainty set to be the smallest hypercube bounding the set which is asymptotically given by the robust set membership identification algorithm given in [15]. This also serves to show the potential improvement in performance which can be achieved by the iterative scheme.

We consider the evolution of the iterative scheme for various desired performance levels, γ_{des}. This is shown in Figures 1 through 4. The smallest partition size is chosen such that the space is split at most three times. The lightly shaded boxes are the ones under current consideration and the dark shaded boxes are the ones that have been invalidated. The values γ correspond to the achievable robust performance for the lightly shaded box, while γ_p corresponds for the performance level achieved when this controller is applied to the actual process. The first execution of the scheme uses $\gamma_{des} = 2.5$ and the results are shown in Figure 1. One can see that the algorithm proceeds towards the box which predicts $\gamma = 1.66$ and the actual performance is satisfied ($\gamma_p = 1.16$). The next example considers the case when $\gamma_{des} = 1.25$. In this case, shown in Figures 2 through 4, the optimistic search leads to the upper right corner of Grid 2 and we see that the actual performance will miss the desired value of 1.25 so the upper right corner is invalidated. In addition, the box (two boxes down from upper right box) having $\gamma = 1.189$ is invalidated because the actual performance misses 1.25. Finally, we come to the box which has $\gamma = 1.249$, and the actual performance is met ($\gamma_p = 1.22$).

When the desired performance is $\gamma = 1.2$, the entire model set is invalidated. To achieve better performance than this one can use the 4th order model. Running the iterative scheme with this model and a desired γ of 1.0, the scheme terminates after 11 iterations with a predicted $\gamma = 0.989$ and the achieved $\gamma_p = 0.987$.

6. Conclusion

This paper presented a new framework for iterative modeling and control. The philosophy of this framework is a very different way of viewing models and their role in designing controllers for uncertain systems. The model is viewed as a tool used to describe the unknown process and really depends on prior information, available control design tools and other modeling preferences. The approach described in this paper was an iterative procedure for refining the uncertainty set via robust control based model invalidation and can be viewed as a systematic way of efficiently searching for a controller delivering a certain desired level of performance to the unknown process. In this way it is possible to invalidate the model if it does not facilitate design of a controller which also provides good performance for the actual process. The result of an iterative scheme in this framework is that either the performance goal will be met or the entire uncertainty set will be invalidated in accordance with our modeling and control method prejudice. An iterative scheme based on a special fixed pole model structure and rank one mixed μ synthesis control design was described in detail and a specific example was used to illustrate the proposed scheme.

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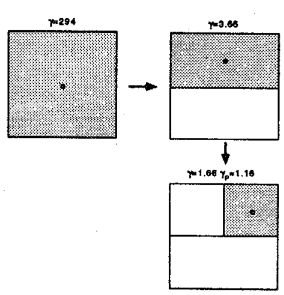

Figure 1: Iterative Scheme ($\gamma_{des} = 2.5$)

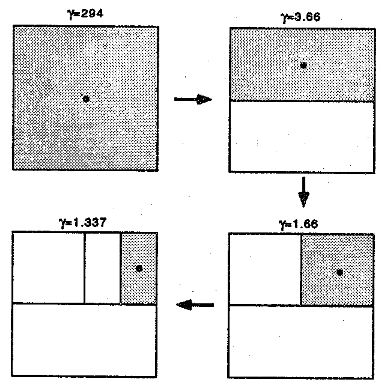

Figure 2: Iterative Scheme: steps 1–4 ($\gamma_{des} = 1.25$)

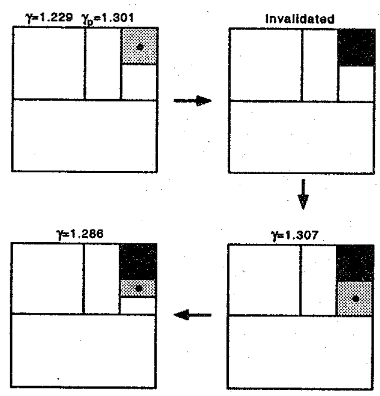

Figure 3: Iterative Scheme: steps 5–8 ($\gamma_{des} = 1.25$)

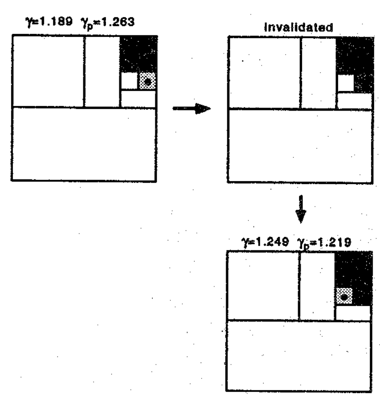

Figure 4: Iterative Scheme: steps 9–12 ($\gamma_{des} = 1.25$)