Abstract—The paper examines path-dependent control of Hybrid Electric Vehicles (HEVs). In this approach we seek to improve HEV fuel economy by optimizing charging and discharging of the vehicle battery depending on the forecasted vehicle route. The route is decomposed into a series connection of route segments with (partially) known properties. The dynamic programming is used as a tool to quantify the benefits offered by route information availability.

I. INTRODUCTION

To reduce fuel consumption, the control of Hybrid Electric Vehicles (HEVs) may be tied to an expected (or to a specified by the driver) traveling route [1], [2]. Utilizing route information, including road characteristics and traffic conditions, the control of the battery charging and discharging can be optimized for a specific route-to-be-traveled. The proliferation of GPS-based navigational systems and digital maps in the modern vehicles facilitates the application of such path dependent control methods for HEVs. Methods to forecast the route to be traveled have been considered in the prior literature, see e.g., [3] and references therein.

The topic of driving condition dependent HEV control has been actively researched in recent years, see e.g., [5], [6], [7], [8] and references therein. Many existing approaches utilize on-line driving pattern recognition to then set accordingly parameters in the control strategy. Other approaches [9] exploit the capability of recurrent neural networks, after appropriate training, to implicitly capture driving pattern information and render control decisions as a single computational algorithm. Dynamic optimization along an anticipated vehicle route has been considered in [10], [4].

Our approach is based on considering the expected fuel consumption over the route as a function of the set-points for battery State of Charge (SoC) in each route segment, expected properties of the route segments and expected characteristics of vehicle speed trajectories in each route segment. Dynamic optimization can then be applied to determine the sequence of the set-points for battery SoC for each route segment. In this paper we illustrate the use of dynamic programming as a dynamic optimization tool, and we quantify the achievable fuel economy benefits that route information availability may offer.

The paper is organized as follows. In Section II the HEV configuration of interest and its existing vehicle control system are briefly reviewed. In Section III, an approach to modeling the fuel consumption during travel over the individual route segments is discussed. Section III describes a dynamic programming approach to the optimization of SoC set-points for the individual route segment. The results of the evaluation of this approach for three simulated routes are reported in Section V. Finally, concluding remarks are made in Section VI.

II. HEV CONFIGURATION AND VEHICLE SYSTEM CONTROL

We consider an HEV configuration shown in Fig. 1. This vehicle is based on a power-split powertrain system, which is similar to the one used in Ford Escape vehicle. The basic components of the HEV are the engine, the battery, a power split device referred to as a planetary gear set, an electric generator, and an electric motor. The planetary gear set splits the power produced by the engine and transfers a part of it to drive the wheels and the rest to the generator to either provide electric power to the motor or to recharge the battery. The engine can provide mechanical power to the wheels and at the same time charge the electric battery through an electric generator, if needed. Depending on the operating conditions, either just the engine or just the electric motor (which consumes electric energy stored in the battery) or both can provide traction power to the wheels to propel the vehicle. The vehicle also incorporates a regenerative braking capability to charge the battery during vehicle deceleration events. Thus the battery can be recharged or discharged by either the electric generator or electric motor or both. Consequently, there are several degrees of freedom in this powertrain configuration to satisfy driver requests. This flexibility can be exploited to optimize fuel consumption.

A Vehicle System Controller (VSC) is used to coordinate subsystems in the HEV. Inherent to this controller is a logical structure to handle various operating modes and a dynamic control strategy associated with each operating mode to specify the vehicle requests to each subsystem. An additional component, called transmission control module (TCM), is used to transmit the controller’s commands to the electric generator and the electric motor. Conceptually, the VSC takes as inputs the environmental conditions, the driver’s requests, and the current state of the vehicle, and provides as outputs the commands for the components, see Fig. 2.
III. FUEL CONSUMPTION MODELING

We consider a route linking an origin \((O)\) to the destination \((D)\), which is decomposed into a series of \(i = 1, \ldots, N\) road segments connected to each other. See Fig. 3.

In each route segment \(i\), of length \(l_i\), the road grade, \(g_i\), and the vehicle speed, \(v_i\), are generally functions of distance and time. The grade is a deterministic quantity which can be known in advance as a function of the distance. With respect to modeling the vehicle speed, in this paper we assume that a nominal vehicle speed trajectory can be predicted for each route segment, possibly dependent on the characteristics of the segment and traffic in the segment.

Our route segmentation criteria generally relate to substantial changes in either the average road grade or average vehicle speed. Such substantial changes in the grade may correspond to the beginning or end of a hill. Such substantial changes for the vehicle speed may coincide with the changes in the road class, decelerations (accelerations) to (from) stop signs or traffic lights, or to traffic jams.

Consequently, a constant average grade, \(g_i\), can be assumed in each segment \(i\). At the same time, a varying nominal vehicle speed trajectory, \(v_i(\cdot)\), has to be considered in each route segment. Such a representative vehicle speed trajectory (a scenario) may be chosen consistently with a finite set of statistical features (mean, variance, etc.) which are considered to be properties of traffic in a particular route segment or type of a driver.

The battery SoC is a key dynamic state in the system. The value of SoC at the beginning of the \(i\)th segment is denoted by \(SoC_i\) and \(SoC_d(i)\) denotes the set-point for the SoC in the \(i\)th segment. The VSC controls the battery SoC in the \(i\)th segment in response to the set-point, \(SoC_d(i)\).

The expected fuel consumption in the \(i\)th route segment is thus a function of \(g_i, v_i(\cdot), l_i, SoC_i\) and \(SoC_d(i)\), i.e.,

\[
\omega_i(g_i, v_i, l_i, SoC_i, SoC_d(i)) = E\left\{ f(g_i, v_i, l_i, SoC_i, SoC_d(i)) \right\}
\]

where \(E\) denotes the expected value. The expectation is used in (1) because the actual vehicle speed trajectory is, in general, not deterministic and can deviate from the nominal trajectory (e.g., due to different driver and traffic situations), and hence the fuel consumption is a random variable.

In our work, we used PSAT (Powertrain System Analysis Toolkit) [12] environment for the HEV simulations. This environment implements both the HEV dynamic model of Ford Escape HEV, and a model of the controller which tracks set-points for battery SoC while satisfying driver requests.

The PSAT model was simulated over segments with different length and grade parameters, with different initial SoCs and SoC set-points and for different vehicle speed trajectories constructed consistently with the chosen feature values as the latter were also varied. A regression model, with the regression terms suggested by the energy analysis of the HEV, and a black box model based on neural network techniques have been developed to fit the collected data set and construct a representation for \(\omega_i\) in (1). An approach where Monte Carlo simulations were employed to average the fuel consumption over several vehicle speed trajectory scenarios has been also implemented. These developments related to fuel consumption modeling from simulated or experimental vehicle data will be considered in more detail in separate publications. The subsequent developments rely on the assumption that a representative fuel consumption model (1) has been developed.
IV. PATH-DEPENDENT CONTROL

A route planner functionality is now described. This functionality prescribes the sequence of SoC set-points, \( \{\text{SoC}_d(i), i = 1, \ldots, N\} \), for the route to minimize the total fuel consumption. The VSC controls the battery SoC in the \( i \)th segment in response to the set-point, \( \text{SoC}_d(i) \).

If we consider a given route as a series of route segments connected to each other with nodes and linking the origin to the destination, then the set-point for battery SoC will be updated at every node and it remains the same as the vehicle travels along a segment of the route. Let \( i \) be the current node and the beginning of the \( i \)th segment of the route, \( i = 1, 2, \ldots, N+1 \), where \( i = 1 \) and \( i = N+1 \) represent, respectively, the origin \( (O) \) and the destination \( (D) \) nodes. The planner incorporates a control law which is a function of the state vector, \( x(i) \), with two components: the segment/node \( i \) and the state of charge SoC\(_i\) at that node. The state dynamics are

\[
x(i+1) = F(x(i), \text{SoC}_d(i)),
\]

\[
x(i) = \begin{pmatrix}
i \\
\text{SoC}_i
\end{pmatrix},
\]

where \( x(i) \) is the state at the current node and \( F \) a nonlinear function, which generates a successor state from the precedent state.

The objective of minimizing the total fuel consumption along the route can be formulated as follows:

\[
\min_{\text{SoC}_d(i)} J = \sum_{i=1}^{N} \omega_i\]

subject to \( \text{SoC}_{\text{min}} \leq \text{SoC}_{i+1} \leq \text{SoC}_{\text{max}} \),

and \( \text{SoC}_{N+1} = \text{SoC}_D \),

where \( J \) is the objective function of our optimization problem, \( \text{SoC}_d(i) (i \in \{1, 2, 3, \ldots, N\}) \) are the manipulated variables, and \( \text{SoC}_{\text{min}}, \text{SoC}_{\text{max}} \) are, respectively, the minimum and maximum limits on SoC. Note that \( J \) is a stage-additive cost function and that the stage cost reflects the expected fuel consumption in each segment \( i \). The constraint \( \text{SoC}_{N+1} = \text{SoC}_D \) is an optional constraint to match SoC to the desired value at the end of the route; the choice \( \text{SoC}_D = \text{SoC}_D \) ensures that the charge is sustained over the traveled route.

In segmenting the route we tend to use segments that are sufficiently long so that feasible \( \text{SoC}_d(i) \) can be tracked within the segment, i.e., \( \text{SoC}_{i+1} = \text{SoC}_d(i) \). In such a case, the dynamics of (4) are simple and the problem complexity is relegated to the fuel consumption model (1). Further, if the fuel consumption can be approximated by a quadratic function of \( \text{SoC}_d \) and \( \text{SoC}_d(i) \), the optimization problem (3) reduces to a quadratic programming problem which can be solved using standard quadratic programming solvers. More general situations can be handled with the dynamic programming as discussed below.

The dynamic programming (DP) translates the property of any final part of an optimal trajectory to be optimal with respect to its initial state into a computational procedure in which the cost-to-go function, \( J^*(x) \), can be recursively computed and satisfies the following relationships:

\[
J^*(x) = \min_{\text{SoC}_d} \{ J^*(F(x, \text{SoC}_d)) + \omega(x, \text{SoC}_d) \},
\]

\[
J^*(x_f) = 0,
\]

where \( \text{SoC}_d = \text{SoC}_d(x_f) \) is the decision variable and, with slight abuse of notations, \( \omega(x, \text{SoC}_d) \) denotes the expected fuel consumption for the state \( x \) and the battery SoC set-point, \( \text{SoC}_d \). At every segment \( i \), the optimal cost \( J^*(x) \) is computed by minimizing over all the sums of the optimal cost-to-go function \( J^*(F(x, \text{SoC}_d)) \) at segment \( i+1 \) plus the cost to move from segment \( i \) to segment \( i+1 \), for all the possible decisions \( \text{SoC}_d \) that can be taken at segment \( i \). Note that the final state in (5) is denoted by \( x_f = x(N+1) \).

Since the model (4) is low dimensional, the effort to numerically compute the DP solution is containable. In the implementation of these computations, the values of SoC and \( \text{SoC}_d \) were quantized so that \( \text{SoC}_d(i) \in \{\text{SoC}^1, \text{SoC}^2, \ldots, \text{SoC}^n\} \) with \( \text{SoC}^1 \leq \text{SoC}^2 \leq \ldots \leq \text{SoC}^n \). Then every node \( i \) may be associated with all possible quantization values, as shown in Fig. 4. As a consequence, the number of all possible values, that the expected fuel consumption for each segment may assume, is equal to the amount of all possible combinations of \( (\text{SoC}_i, \text{SoC}_d(i)) \), with \( \text{SoC}_i \) and \( \text{SoC}_d(i) \) quantized. The number of all these possible combinations is \( n^2 \) and thus the expected fuel consumption can take \( n^2 \) different values \( \{\omega^1, \omega^2, \ldots, \omega^{n^2}\} \), for a given route segment.

V. RESULTS

To quantify the potential benefits of the route-dependent control, we consider several case studies. In these case studies, the grade and the vehicle speed trajectory in each segment were assumed to be known. The expected fuel consumption was, therefore, a deterministic quantity, and no
TABLE I
FUEL CONSUMPTION FOR A ZERO-GRADE ROUTE.

| FUEL SAVINGS | Total fuel consumption \( (kg) \) | \( SoC_d \) sequence | \( (\%) \) |
|-------------|----------------|-------------------|
| No SoC control | 0.37 | 50-50-50-50-50-50-50-50 |
| DP SoC control | 0.32 | 50-52-50-48-46-46-44-50 |

averaging with respect to random realizations of the vehicle speed trajectory was employed.

A. Route With Zero Grade

Our first route is shown in Fig. 5, where \( O \) the origin and \( D \) is the destination. The route was decomposed into \( N = 7 \) segments. Length and grade information for each road segment and the vehicle speed trajectory in each segment were assumed to be available and known in advance.

![Fig. 5. Example route - Zero grade.](image)

For the route in Fig. 5, the grade was assumed to be zero and the \( SoC \) of the vehicle at the origin is \( SoC_O = 50\% \). To sustain the charge in the battery, the desired \( SoC \) at the destination node is equal to \( SoC_D = 50\% \). The values of \( SoC_{\text{min}} \) and \( SoC_{\text{max}} \) were set to 40\% and 60\%, respectively.

Table I compares the fuel consumption with \( SoC_d \) prescribed by the DP policy, which we refer to as “DP SoC Control” case, and the fuel consumption when \( SoC_d(i) = 50\% \) in each segment, which we refer to as “No SoC control” case. In the former case, the fuel consumption (0.32 kg) is about 13.5\% lower than in the latter case (0.37 kg). This represents a significant improvement.

We note that our route segmentation in Fig. 5 is not based on using segments of equal length or equal travel duration, but rather on the available vehicle speed information. Specifically, the nodes when one segment ends and another begins (and where SoC control points are located) correspond to the initiation of a significant change in average vehicle speed. The nominal vehicle speed trajectory was constructed so that in each segment, a constant rate acceleration or deceleration to the new vehicle speed value is assumed, followed by steady cruise at that speed.

B. Route with Non-Zero Grade

A grade of 5\% was inserted at segment 2 of the route in Fig. 5, while the rest of the route characteristics remain unchanged. See Figure 6.

![Fig. 6. Example route - Non-zero grade.](image)

Table II compares the fuel consumption in “No SoC control” case with fuel consumption in “DP SoC control with grade ignored” case and “DP SoC control with grade included” case. The second case employs the same SoC setpoints, \( SoC_d(i) \), as in Table I, i.e., it is the case in which only vehicle speed information and no road grade information has been taken into account in the optimization.

Compared with the previous case study, the total fuel consumption in the case of “No SoC control” has increased from 0.37kg to 0.4kg. This increase may be explained by the presence of a large uphill grade on segment 2. The fuel consumption in “DP SoC control with grade ignored” case is 7.5\% less. By including the grade information into the DP optimization, a further decrease in fuel consumption, by additional 2.7\%, is effected.

Similarly to the case of speed information, grade information should also constitute a route segmentation criterion. In particular, a significant change in the average grade of the route may prescribe the beginning of a new segment and an additional SoC control point.

C. Route With Larger Number of Segments

An urban route from Boston, MA, shown in Fig. 7, has been selected as another case study. The urban environment
of the chosen route includes roads of different speed classes, several traffic lights and stop signs, which all result in frequent and significant speed changes. At the same time, this is a relatively flat route, in the sense that the road grade varies within a relatively small range, $[-1.5\%, 1\%]$. The route was segmented into 34 segments to reflect significant changes in the vehicle speed (including decelerations to stop signs/traffic lights, and accelerations from stop/traffic lights) and changes in the road grade.

![Real route, Boston, MA](image)

A nominal vehicle speed trajectory was constructed as follows. From each stop, the vehicle accelerates to a steady maximum speed at a constant acceleration rate. The vehicle decelerates to zero at a constant deceleration rate. The steady travel speeds and acceleration and deceleration rates were matched to observed values along the route in light traffic conditions. The vehicle was assumed to stop at all stop signs and traffic lights, consistently with observations of an actual vehicle driving this route.

The fuel consumption in the “DP SoC Control with grade included” case is approximately 4.8% less over this route than in the “No SoC control” case. Larger benefits are anticipated if this route had larger grades.

VI. CONCLUSIONS

In this paper, we described an approach to controlling Hybrid Electric Vehicles so that to reduce their fuel consumption along a known or predicted path. The approach aims to incorporate information about traveled route and traffic, which may be readily available to future vehicles. Specifically, an algorithm based on Dynamic Programming, for SoC set-point optimization along the route was proposed. Its application demonstrated a potential for fuel economy improvements, with the level of benefits dependent on a specific route being traveled.

Certain approaches for segmenting a route have been discussed. They generally relate to significant changes in average vehicle speed or road grade and to the presence of stop signs/traffic lights. With this segmentation approach, the resulting segments do not necessarily have the same length or travel time. Research is presently on-going to understand the effects of granularity/accuracy of the information regarding the route segment properties on the fuel consumption reduction benefits.

Several extensions of the problem formulation, which may be treated similarly, include advising the driver on the vehicle speed to maintain along a known/predicted route. Advising the driver on the route to take to reduce fuel consumption with acceptable degradation of travel time may also be considered.

VII. ACKNOWLEDGMENTS

The authors would like to thank Yanan Zhao and Tim Feldkamp of Ford Motor Company for their contributions to fuel consumption modeling aspects of this research.

REFERENCES