Optimal Control of the DISC Engine using Hierarchical and Quantized Control

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Abstract— **In this paper, we present an approach for controlling switched systems that reduces the complexity of determining both a control input and mode sequence to a dynamic programming problem in finding an optimal set of states to track in each operating mode, using custom controllers in each mode to perform such tracking. The results are focused on an application to the direct injection stratified charge (DISC) engine, which uses two modes of operation to provide a tradeoff between power and fuel economy. In this application, a new type of torque and air-to-fuel ratio controller, based on multi-parametric quadratic programming, is utilized.**

I. Introduction

The increasing complexity of modern gasoline engines as a means to providing the flexibility required for improving fuel economy while preserving performance has resulted in a need for controllers that can effectively and efficiently control such systems. The direct injection stratified charge (DISC) engine is an example of a modern, complex engine where by the complexity in control lies in the inclusion of two operating modes (homogenous and stratified) that provide tradeoffs in fuel economy, power output, and emissions.

In homogenous operation, fuel is injected during the intake stroke, providing an approximately even air-fuel mixture throughout the cylinder. The characteristics of the engine are similar to that of the typical port-fuel injection (PFI) engine in terms of performance and emissions, and the air-to-fuel ratio (AFR) operates about the stoichiometric value of 14.6:1.

In the stratified operation, fuel is injected late into the compression stroke, forcing the fuel, under the influence of a specialized piston head, to be concentrated about the spark plug. The typical AFR for this mode of operation is about 35:1, significantly higher than that of the PFI engine. However, this operation regime also generates a significant amount of NO_x byproduct that must be accounted for by a specialized catalyst which, over time, can lose its effectiveness and must be regenerated by switching the engine for some time back into the homogenous regime.

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This work was supported in part by the Ford Research Laboratory

The combustion mode to apply depends upon the amount of torque demanded by the driver and catalyst's state. When torque demands are high, the engine should be operated in homogenous mode. When the demanded torque is low to moderate, and the catalyst is operating effectively, stratified operation should be used to increase fuel economy. Ultimately, a high-level controller uses the torque demanded by the driver and the catalyst state to compute the appropriate combustion mode as well as a set point for the intake-manifold pressure and the AFR. The purpose of the lower-level DISC engine controller is to track these references so as to guarantee convergence to these set-points as well as a desired level of performance.

In this paper, a new controller for controlling the AFR and torque as a function of the intake-manifold pressure and combustion mode is introduced as well as a novel approach for reducing the complexity of controllers designed for constrained switched-systems such as the DISC engine.

II. Modeling and Control Objectives of the DISC Engine

A. Overview

In this paper, we treat the simplified model of the DISC engine presented in [2], a derivative of the mean-value model proposed and verified in [7]. Below, we present a brief overview of the DISC engine parameters and present the form of the linearized, discrete-time DISC engine model.

B. Model Parameters

DISC engine set-points are determined by a high-level controller that, based upon the torque demanded by the driver and the state of the catalyst, computes a nominal reference point for several engine parameters. The tracking parameters are:

- Intake-manifold pressure (IMP) p_m, which governs the mass-flow rate of air into the cylinder and impacts the air-to-fuel ratio (AFR).
- Air-to-fuel ratio λ, which is the ratio of the mass flow rates of air and fuel into the cylinder and is a determining factor for fuel-economy.
- Brake torque τ, the torque provided to the driver, which is the sum of the indicated torque (torque generated through combustion) and frictional losses.

- Combustion mode ρ that determines which operating mode to use: stratified ($\rho = 1$) or homogenous ($\rho = 2$).

The following parameters are treated as control inputs to the system:

- Mass flow rate of air through the throttle W_{th} that controls p_m.
- Fueling rate W_f that affects both the AFR and the brake torque.
- Spark timing δ that impacts the amount of generated torque.
- Combustion mode ρ.

It is not desirable to simply switch operating modes when the reference combustion mode changes because the change in system dynamics may have a significant impact on the output parameters. The problem of determining how to choose a value for ρ at each time sample is the subject of section 4.

There is a particular spark timing, termed the *maximum brake torque (MBT) spark timing* δ_{mbt}, that provides maximum torque to the engine and is an affine function of λ that depends on ρ. As δ deviates from this parameter, the indicated torque decreases quadratically.

C. Constraints

In both operating modes, there exist actuator saturations and other practical limitations that are treated as hard constraints on the ranges of W_{th}, W_f, and δ. To simplify control, we follow convention and restrict the spark timing to the interval $[0, \delta_{mbt}]$.

To avoid misfirings and excessive emissions caused by either too rich or too lean an air-fuel mixture, the output parameter λ is specially bounded to a range that depends on the combustion regime:

$$\lambda_{\min}(\rho) \le \lambda \le \lambda_{\max}(\rho) \tag{1}$$

The remaining output parameters p_m and τ are naturally bounded by the above limitations.

D. The Linear Model

The form of the linear, discrete-time DISC engine model is:

$$\begin{gathered} \hat{p}_m(n+1) = a(\rho)\hat{p}_m(n) + \begin{bmatrix} b(\rho) & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{W}_{th}(n) \\ \hat{W}_f(n) \\ \hat{\delta}(n) \end{bmatrix} \\ \begin{bmatrix} \hat{p}_m(n) \\ \hat{\lambda}(n) \\ \hat{\tau}(n) \end{bmatrix} = \begin{bmatrix} 1 \\ c_2(\rho) \\ c_3(\rho) \end{bmatrix} \hat{p}_m(n) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & d_{22}(\rho) & d_{23}(\rho) \\ 0 & d_{32}(\rho) & d_{33}(\rho) \end{bmatrix} \begin{bmatrix} \hat{W}_{th}(n) \\ \hat{W}_f(n) \\ \hat{\delta}(n) \end{bmatrix} \end{gathered} \tag{2}$$

where $\hat{v}$ denotes the deviation of the parameter v from its value at the operating point in the corresponding combustion mode.

In the sequel, we will refer to the input vector as $u(n)$, the state as $x(n)$, the output as $y(n)$, and the system matrices as A,B,C,D.

III. Single-Mode Controller

The objective of the single-mode controller is to optimally track (in a fixed mode of operation) a reference output y_{ref} according to the cost function:

$$J = \sum_{n=0}^{\infty} \left(y_{ref} - y(n)\right)^T Q \left(y_{ref} - y(n)\right) \tag{3}$$

where $y(n)$ is the system output and Q is a positive definite matrix. It is assumed that y_{ref} is *achievable*, meaning that there exists an input that can track this output in steady state in mode ρ. Hard constraints on the control signals naturally bound the actuators.

A. Open-Loop Single-Mode Controller

There exists a nice separation between the input that affects the state and the inputs that affect the output in (2) that, under the assumption that all references are achievable, allows us to optimally track a reference with respect to (3) simply by tracking the IMP in minimum time and minimizing the error of the AFR and the torque at every time step.

To optimally determine values for the inputs $W_f(n)$ and $\delta(n)$ so as to minimize the errors in λ and τ while satisfying hard constraints, we use a quadratic program (QP) that depends upon the plant's state and the reference.

To track the state in minimum time, we implement a basic discrete-time bang-bang controller for determining $W_{th}(n)$:

$$\hat{W}_{th}(n+1) = sat\left(\frac{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \hat{y}_{ref} - a(\rho)\hat{x}(n)}{b(\rho)} \right) \tag{4}$$

where $sat(\cdot)$ is the saturation function.

The controller for determining $W_f(n)$ and $\delta(n)$ is a QP that seeks to minimize the error in λ and τ while meeting hard constraints on the inputs and the outputs. Given a reference output y_{ref} and combustion mode ρ, let

$$\left.\begin{aligned} y_{23} &= \begin{bmatrix} \lambda & \tau \end{bmatrix}^T \\ Q_{23} &= PQP^T \\ y_{23}^{ref} &= Py_{ref} \end{aligned}\;\right|\; \begin{aligned} u_{23} &= \begin{bmatrix} W_f & \delta \end{bmatrix}^T \\ D_{23}(\rho) &= PD(\rho)P^T \\ C_{23}(\rho) &= PC(\rho)P^T \end{aligned}$$

where $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

The optimizing quadratic program is:

$$
\begin{aligned}
&\min \quad \left(y_{23}^{ref} - y_{23}\right)^T Q_{23}\left(y_{23}^{ref} - y_{23}\right) \\
&\text{subj. to} \\
&\qquad \begin{bmatrix} \lambda_{\min}(\rho) \\ \tau_{\min} \end{bmatrix} \le y_{23} \le \begin{bmatrix} \lambda_{\max}(\rho) \\ \tau_{\max} \end{bmatrix} \\
&\qquad \begin{bmatrix} W_{f,\min} \\ 0 \end{bmatrix} \le u_{23} \le \begin{bmatrix} W_{f,\max} \\ \delta_{mbt} \end{bmatrix} \\
&\text{where} \\
&\qquad \delta_{mbt} = k_{\delta 1}(\rho) + k_{\delta 2}(\rho)\lambda \\
&\qquad \hat{u}_{23} = \left(D_{23}(\rho)\right)^{-1}\left(\hat{y}_{23} - C_{23}(\rho)\hat{x}(n)\right)
\end{aligned}
\tag{5}
$$

The above QP determines the optimal value for y_{23}, which can then be used to compute the control input u_{23}. However, solving (5) at every time sample of a fast-paced system such as the DISC engine is not practical, and so we employ the multi-parametric quadratic programming (MP-QP) techniques detailed in [3]. Approximately 20 partitions in the parameter space are required to store the exact solution (5) in memory.

B. *Closed-Loop Controller*

Of course, even if multiple linearizations are utilized, the controller is still open-loop, so zero-offset tracking cannot be guaranteed. Integral action is applied to (5) to obtain a closed-loop controller for λ and τ.

$$
\begin{aligned}
&1.\ e \leftarrow (1-\alpha)\left(y_{23}^{ref} - y_{23}(n)\right) + e_I(n-1) \\
&2.\ \text{Let } y'_{23} \text{ be the solution to (5) applying the} \\
&\quad \text{reference } \tilde{y}_{23}^{ref} = y_{23}^{ref} + e. \\
&3.\ e_I(n) \leftarrow y'_{23} - y_{23}^{ref} \\
&4.\ \hat{u}_{23} = \left(D_{23}(\rho)\right)^{-1}\left(\hat{y}'_{23} - C_{23}(\rho)\hat{x}(n)\right)
\end{aligned}
\tag{6}
$$

where the constant $0 < \alpha < 1$ determines the trade off between the convergence rate and sensitivity to noise. It can be shown that (6) asymptotically stabilizes λ and τ to their optimal values with respect to a fixed mode of operation and system state. We refer the reader to [9] for the proof of this result as well as a comparison of (6) to another controller designed for the same application.

C. *Approximating via Multiple Linearizations*

It is shown in [9] how to adapt (4) and (6) for use with multiple linearizations of the nonlinear DISC engine model. For brevity, we do not discuss these approaches here as they are not relevant, but we note that the simulations in section 5 make use of such multiple linearizations.

IV. Reducing the Complexity in Switched Control

A. *Motivation*

The remaining difficulty in controlling the DISC engine lies in determining the control input sequence $\rho(n)$ for the operating mode. Computing an optimal control input and switching sequence for a constrained switched system is classified as NP-Hard [1], and so we seek approaches that strive for sub-optimality with guarantees in convergence and performance.

The use of model-predictive control (MPC) has been proposed as one potential means for handling a large array of hybrid-control problems, including switched-system control, by reducing the number of controller decisions to a short look-ahead horizon [1]. However, in an application of MPC to the DISC engine, the feasibility of the quadratic program used to compute the control law could only be guaranteed by softening the hard constraints on the actuators [2], which may negatively impact convergence.

It was shown in [4] and [5] that the use of hierarchal control combined with a quantization in the controller decisions can greatly reduce computational complexity, allowing for the use of planning algorithms that guarantee convergence and quality of performance. We seek to apply the same methodology to the switched system of the DISC engine. Our method is different from the approach in [6] in that we do not approximate the optimal cost-to-go function to compute, in real-time, an approximation to the optimal control input. We derive a switching sequence by examining the cost-to-go between switching states using predesigned controllers in each operating mode.

B. *Constructing the Robust Hybrid Switching Graph*

Let $M \subset Z^+$ be the finite set of operating modes for a controlled switched system ($M = \{1,2\}$ for the DISC engine). Term a parameter *admissible* if it satisfies its boundary constraints. We define X_m, Y_m, and U_m as the admissible sets for the state, output, and input respectively in mode $m \in M$. Define X, Y, and U respectively, as the union of X_m, Y_m, and U_m over all operating modes.

Define the *switchable set* between two operation modes m and n in M as those admissible states that are common to both modes:

$$X_{mn} = X_m \cap X_n \tag{7}$$

Assume that X_{mn} is bounded for all $m,n \in M$. We will now construct sets S_{mn} of switching states between pairs of modes:

$$
\begin{aligned}
&1.\ S_{mn} \subset X_{mn} \\
&2.\ x_1, x_2 \in S_{mn},\ x_1 \ne x_2 \Rightarrow 0 < 2r_s < \left\|x_1 - x_2\right\|_2
\end{aligned}
\tag{8}
$$

Essentially, the set of switching states represents those states the controller must track in order to switch from one mode to another. Once the system's state is within the *switching radius* r_s of one of switching states in S_{mn}, the system may switch between these modes. The switching radius should be small but not so small that tracking it is unnecessarily difficult (from the effects of noise, for example). Clearly, by the assumption that X_{mn} is bounded, the set of switching states is finite.

For each switching state, we define a *switching region* as $SR(x) = \{x_r \mid \|x_r - x\|_2 < r_s\}$. The set of all switching states of mode $m \in M$ is given as $S_m = \bigcup_{n \in M} S_{mn}$.

We now construct the *robust hybrid switching graph* (RHSG) for the system. Define the directed RHSG $G = (V, E)$ by:

$$
\begin{aligned}
&1.\ v = (m, x) \in V \Leftrightarrow x \in S_m \\
&2.\ (v_1, v_2) = ((m, x_1), (n, x_2)) \in E \Leftrightarrow \\
&\qquad \{(m = n) \text{ and } (x_1, x_2 \in S_m)\} \\
&\qquad \text{or } \{(m \neq n) \text{ and } (x_1 = x_2 \in S_{mn})\}
\end{aligned} \tag{9}
$$

The first condition simply states that all the switching states are vertices in the RHSG and vice-versa. The second condition states there exist edges between all the switching states in the same mode, and between the same switching state in two different modes.

C. Applying the RHSG to Determining the Optimal Switching Path

Assume that for each mode *m* of the system there exists a control law $u_m \in U_m$ such that for any initial state $x_0 \in X_m$, final state $x_d \in X_m$, and a reference $y_{ref} \in Y$, there exists a time $T \geq 0$ so that $x(T) = x_d$. Furthermore, assume u_m and T are chosen to minimize

$$
J_m = \int_0^T L(g_m(x, u) - y_{ref}, u_m) dt \tag{10}
$$

where L is a positive definite function, and g_m is the output of the system while in mode *m*. Denote $J_m^*(x_0, x_d, y_{ref})$ as the corresponding optimal cost. Also, let $\hat{u}_m \in U_m$ be the control law that minimizes (10) for $T = \infty$ and denote the corresponding optimal cost as $\hat{J}_m^*(x_0, x_d, y_{ref})$.

By the limitation of only being able to choose from a finite set of switching states, and by having a control law that can track these states in finite time, it is only necessary to determine the sequence of switching states to track (termed the *switching path*) from an initial state x_0 in mode p to a reference state x_{ref} (corresponding to the reference output y_{ref}) in mode q.

The RHSG graph *G* is appended with additional edges connecting the switching states of the initial and reference modes to, respectively, the initial and reference states. Let $v_0 = (p, x_0)$ and $v_f = (q, x_{ref})$, and define the directed, weighted graph $G' = (V', E')$, called the *appended RHSG*, by:

$$
\begin{aligned}
&1.\ V' = V \cup \{v_0, v_f\} \\
&2.\ E' = E \cup \left\{ \bigcup_{x \in S_p} (v_0, (p, x)) \right\} \cup \left\{ \bigcup_{x \in S_q} ((q, x), v_f) \right\}
\end{aligned} \tag{11}
$$

Define the weighting function $w : E' \to \Re$ on G' as:

$$
w(e) = \begin{cases} \varepsilon_s & , m_1 \neq m_2 \\ J_{m_1}^*(x_1, x_2, y_{ref}), & m_1 = m_2 \text{ and } x_2 \neq x_{ref} \\ \hat{J}_{m_1}^*(x_1, x_2, y_{ref}), & \text{otherwise} \end{cases} \tag{12}
$$

where ε_s is the switching penalty. Essentially, unless the next state in the switching path is the reference state, it should be tracked in finite time. Otherwise, the infinite-horizon optimal control law is applied.

Subject to the above constraints, we can now determine the optimal switching path simply by finding the "shortest" path (the path of least cost) from v_0 to v_f in G', an algorithm with $O(|E'| \log_2 |V'|)$ complexity.

Let the function $SPN : V' \to V'$ be a mapping of a node in the graph G' to the next node in the optimal switching path and define $SPN(v_f) = v_f$. The optimal switching path starting from v_0 can be then be written as $P = [SPN(v_0), SPN(SPN(v_0)), \ldots] = [p_1, p_2, \ldots]$ and the number of switches as $N = \min\{i \mid p_i = p_j, j > i\}$. Since the shortest-path in the appended RHSG contains no cycles, *N* is guaranteed to be less than $|V'|$, meaning there are only a finite number of switches in the switching path.

It is possible that a controller which minimizes the error with respect y_{ref} while tracking x_d in finite time may be too computationally intensive to use. As an alternative, one may apply a controller that optimally tracks a different reference, $\hat{y}_{ref}$, that is achievable in steady-state at x_d and optimal with respect to y_{ref}. Of course, the resulting full controller will not be optimal in the traditional *p*-norm sense, but the optimization may still meaningful and potentially worth the reduction in complexity.

D. Making RHSG Applicable to High-Speed Processes

It may not be practical to compute the costs between large numbers of vertices or, perhaps, to even apply a shortest-path algorithm on every time step of the system, and so a means for storing a rough approximation to the solution of the dynamic program in memory is required.

Given the granularity of the RHSG shortest-path problem, we can assume that small shifts in the initial and reference states do not impact the switching path before the reference state. Therefore, we can quantize the state and reference output spaces and expect a fairly good approximation to the exact switching path.

Define the set $\tilde{X}_m$, the set of *approximation points* in the admissible set of the state space in operating mode *m*, by:

$$
\begin{aligned}
&1.\ S_m \subset \tilde{X}_m \subset X_m \\
&2.\ x_1, x_2 \in \tilde{X}_m,\ x_1 \neq x_2 \Rightarrow 2r_s < \|x_1 - x_2\|_2 \\
&3.\ \tilde{X}_m \text{ is finite}
\end{aligned} \tag{13}
$$

The first condition states that the switching states also act as approximation points. The second condition ensures that a switching state is the approximation point for its switching region, which is necessary for convergence. Define the state approximation function $APX_m : X_m \to \widetilde{X}_m$ as $APX_m(x) = \arg\min_{z \in \widetilde{X}_m} \{\|x - z\|_2\}$.

Similarly, define $\widetilde{Y}_m$ and APY_m for the reference space (though the distance between such approximation points may be any positive constant).

Given a finite number of approximation points in both spaces, we can now store the solutions in a table. For an initial mode *m* and reference mode *n*, define the *RHSG table* $T_{mn} : \widetilde{X}_m \times \widetilde{Y}_n \to M \times X$ as:

$$T_{mn}(\widetilde{x}, \widetilde{y}) = SPN((m, \widetilde{x})) \text{ (with } v_0 = (m, \widetilde{x}) \text{ and } v_f = (n, \widetilde{y})) \tag{14}$$

E. The Full Controller

We apply the RHSG table to the feedback control laws for each mode *m* in an open-loop fashion. Table lookups are only performed when the mode or the reference changes, otherwise the system drives the plant's state to the next state in the switching path determined by the previous lookup. Denote the reference state, output, and mode, respectively, as x_{ref}, y_{ref}, and *q*, and let $x(n)$, $y(n)$, and $\rho(n)$ be the state, output, and operating mode of the plant at time sample *n*. The following algorithm uses the RHSG table to guide the plant along the optimal switching path:

1. if $x(n) \in SR(x_d)$ then $\rho(n+1) \leftarrow m_{next}$ else $\rho(n+1) \leftarrow \rho(n)$
2. if *either the reference changes* or $\rho(n) \neq \rho(n+1)$ then
3. $\quad \widetilde{x} \leftarrow APX_{\rho(n+1)}(x(n))$
4. $\quad (m_{next}, x_d) \leftarrow T_{\rho(n+1)q}(\widetilde{x}, APY_q(y_{ref}))$
5. $\quad$ if $\rho(n+1) = m_{next}$ then $x_d \leftarrow x_{ref}$
6. end
7. $u(n+1) \leftarrow \begin{cases} \hat{u}_{\rho(n+1)}(x(n), y(n), x_d, y_{ref}), & x_d = x_{ref} \\ u_{\rho(n+1)}(x(n), y(n), x_d, y_{ref}) & \text{otherwise} \end{cases}$

(15)

The system makes a switch (line 1) when it the state lies in the switching region of the switching state x_d. Whenever a switch occurs (line 2), the next state in the switching path is referenced (line 4) and tracked until the next switch. When the remainder of the switching path is constant, that path is exhausted, and so the true reference is tracked (line 5). By the assumptions on the control laws and the finiteness of the switching path, (15) is guaranteed to converge to reference, though not in the traditional sense of asymptotic stability due to the limitation of using only a fixed set of switching points.

V. Applying RHSG to the DISC Engine

A. Full Controller for the DISC Engine

We apply (15) using the control law constructed from (4) and (6) to build the full controller for the DISC engine.

We modify (4) so that the state x_d provided by the switching path is tracked:

$$\hat{W}_{th}(n) = sat\left(\frac{\hat{x}_d - a(\rho)\hat{x}(n-1)}{b(\rho)}\right) \tag{16}$$

Algorithm (6) remains unchanged for determining the values of $W_f(n)$ and $\delta(n)$ as it minimizes the error between the reference and the output AFR and torque.

B. Reducing the RHSG Table Size

Although usable RHSG tables of practical sizes can easily be generated for the DISC engine, we briefly consider in this section a means for significantly reducing the table's memory requirements for systems possessing scalar states. Consider the rules below:

- *Rule A*: If, for a given state x_0 and reference y_{ref}, the tracking state is x_d, then the same tracking state should be used for all states in between x_0 and x_d.

- *Rule B*: The operating mode may change only once when tracking a reference.

Applying these assumptions, the RHSG table format may be modified. For each reference approximation point $\widetilde{y}_{ref} \in \widetilde{Y}_\rho$ in reference mode ρ, the following data structure may be used:

$$T_D(\widetilde{y}_{ref}, \rho) = \begin{bmatrix} -\infty & x_{l2} & x_{l3} & \cdots & x_{ln} \\ x_{s1} & x_{s2} & x_{s3} & \cdots & x_{sn} \end{bmatrix} \tag{17}$$

By Rule B, it is redundant to store a table for the case that the state and reference are in the same mode, so table (17) assumes that the system is in a different mode than the reference. The first row specifies the range of points that drive to a switching state, the second row is the switching states to use. The algorithm for getting the next switching state is:

1. $T \leftarrow T_D(APY_\rho(y_{ref}), \rho)$
2. $i \leftarrow \max\{j \mid x(n) \geq [T]_{1,j}\}$
3. $x_d \leftarrow [T]_{2,i}$

(18)

C. Simulation Results

The simulation results in Figure 1 were obtained using a 22KB table with the following attributes: 106 switching points along in the IMP, 49 and 36 approximation points along the IMP in the stratified and homogenous modes respectively, and 48 and 29 approximation points in the reference output space in the stratified and homogenous modes respectively. It took roughly 2 hours to generate the table using a 1GHz PC.

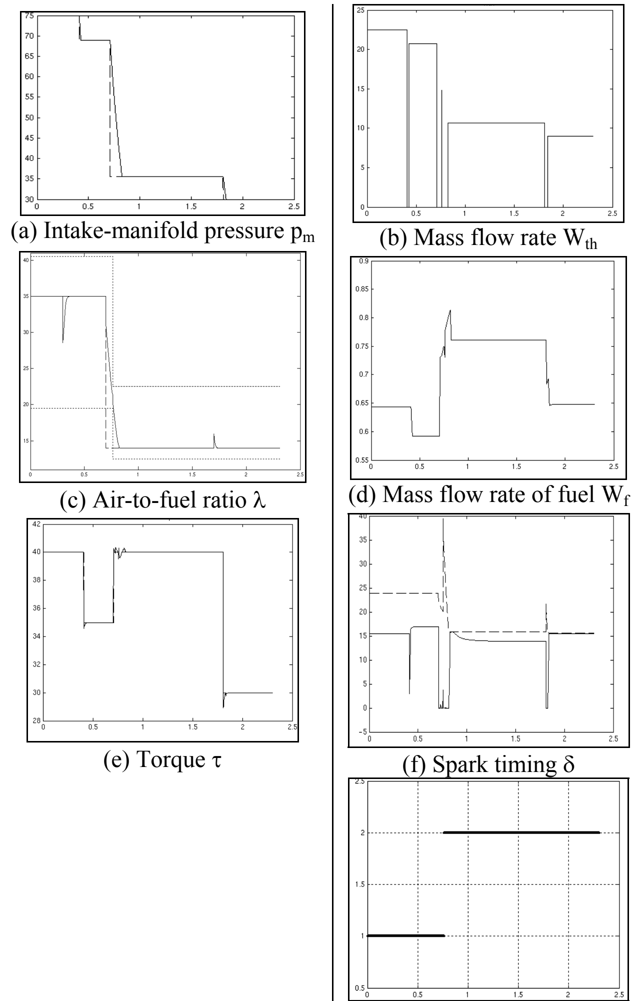
The left column of Figure 1 shows the system's responses, and the right column shows the control inputs. The system is initialized in the stratified regime, and tracks the IMP, AFR, and torque references. The use of MP-QP allowed for a very fast tracking of the latter two parameters while satisfying the hard constraints on the inputs.

When the reference indicates switching to the homogenous regime (at the time when the reference AFR drops to roughly 14:1), the controller examines the RHSG table and selects a switching point along the IMP (in this case, the switching point is at 52 kPa). When the pressure reaches this point, the system switches modes, and the controller drives to the reference IMP. The mode shift causes a change in the system dynamics, and, consequently, a perturbation in the torque and AFR. This disturbance, however, is relatively insignificant, meaning the RHSG shortest-path algorithm chose a good switching point.

VI. CONCLUSIONS

In this paper, we presented a new approach to the control of switched systems that relied on reducing the NP-Hard problem of determining both an optimal control and mode sequence to a simpler dynamic program requiring the determination of switching states based in the use of fixed controllers in each operating mode. Because solving the RHSG shortest-path problem online may not be practical, a simple quantization of the admissible portions of the state the reference spaces are used to construct a table that can be quickly referenced online.

The RHSG approach is applied in conjunction with a new type of AFR and torque controller for the DISC engine that allows for the fast tracking of achievable references. The structure of the DISC engine model was such that a modified version of the RHSG table, using far less memory, could be constructed. Simulations verified that the full controller effectively selected and tracked switching paths to the reference.



(a) Intake-manifold pressure p_m

(b) Mass flow rate W_{th}

(c) Air-to-fuel ratio λ

(d) Mass flow rate of fuel W_f

(e) Torque τ

(f) Spark timing δ

(g) Combustion regime

Figure 1(a)-(g): Simulation results for the full controller applied to the nonlinear DISC engine. Solid line – response of the system; Dashed line – reference (except for spark timing where dashed line is MBT spark timing. The fine-dotted lines in AFR represent the AFR boundaries).

VII. ACKNOWLEDGEMENTS

The authors would like to thank Davor Hrovat of Ford Research Laboratory for initiating and participating in this collaboration.

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